

ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Departamento de Engenharia Mecânica

Trabalho de Formatura

ESTUDO DE UMA ESTRUTURA TIPO GRELHA

Nota: 9,0 (nove)

20/11/1980

Ferreira

Autor: Heraldo Horta Rodrigues

Orientador: Prof. Waldo Perseu Augusto Pereira

1980

À minha esposa

Eliana,
e minhas filhas
Ana Cláudia
Mariana

ÍNDICE

I-	Introdução.....	1
II-	Introdução Teórica.....	3
III-	Modelo Teórico das Vigas.....	8
IV-	Cálculo dos Sistemas.....	10
	Caso A.....	10
	Caso B.....	20
	Caso C.....	30
	Caso D.....	40
V-	Soluções dos Sistemas.....	47
VI-	Conclusões.....	56
VII-	Referências Bibliográficas.....	62
VIII-	Anexos.....	63

I - INTRODUÇÃO

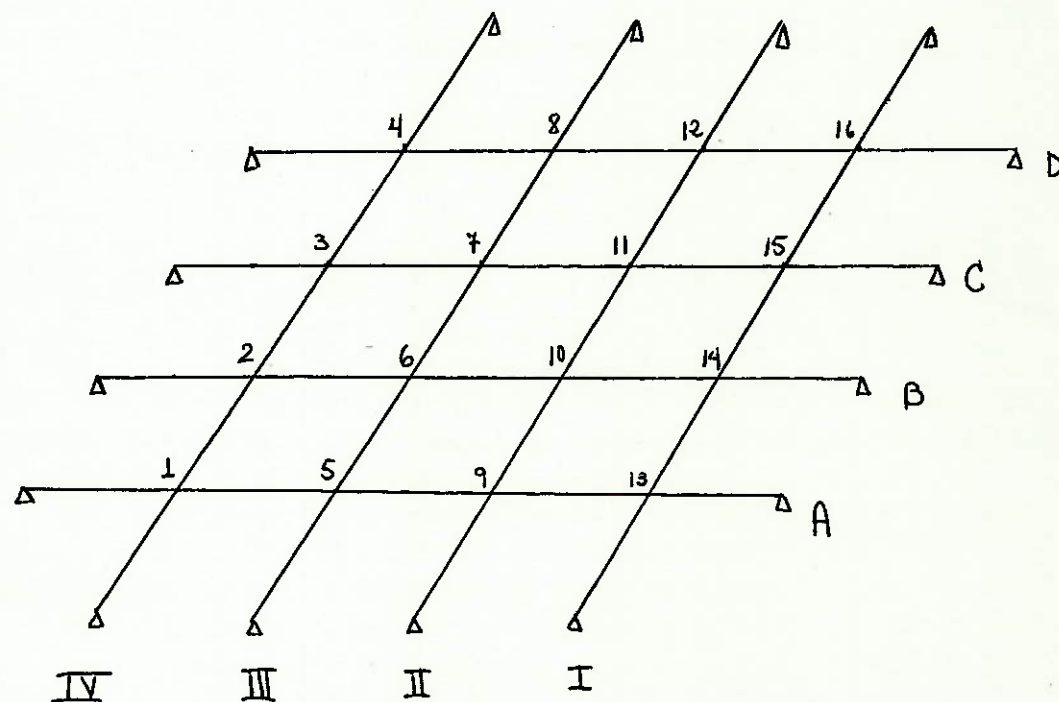
O modelo matemático se compõe da estrutura quadriculada abaixo, supondo as vigas simplesmente apoiadas nas extremidades.

Supõe-se também as vigas A, B, C, D superpostas e apoiadas às vigas I, II, III, IV, sendo que estão ligadas entre si nos nós.

O objetivo do presente é elaborar um estudo para verificar o comportamento das reações em cada barra, quando se aplicam carregamentos na estrutura.

Para estudar os esforços que, aparecem, quando se aplicam cargas sobre as barras, podemos dividir o problema em 4 casos.

Iremos aplicar esforços unitários em 4 posições chaves, para avaliar as reações subsequentes.



Outra hipótese a considerar é a de que todas as barras são de secção constante e de mesmo Momento de Inércia.

Em outras palavras , estamos supondo todas as barras com $L/EI = \text{cte}$.

As posições chaves são os nós 1 , 2 , 5 , 6 , tendo em vista a simetria da estrutura.

Dividiremos o problema em 4 casos:

CASO A - esforço unitário aplicado ao nó 1. Com isso estudaremos as flechas em todas as vigas , em todos os nós, e pela igualdade das flechas em cada nó , obteremos um sistema de 17 equações a 17 incógnitas .

O mesmo estudo será feito para os nós 5 , 6 , 2 , constituindo os casos B , C , D .

As diversas combinações possíveis desses 4 casos nos permitirão determinar o comportamento geral da estrutura , para cargas unitárias atuando isoladas ou conjuntamente , desde que aplicadas nos nós ou fora deles, mas sobre as barras , admitindo-se , a hipótese de que uma carga fora do nó tenha a distribuição de seu efeito repartida apenas entre os dois nós contíguos.

O problema não abrange o caso de cargas aplicadas fora das barras .

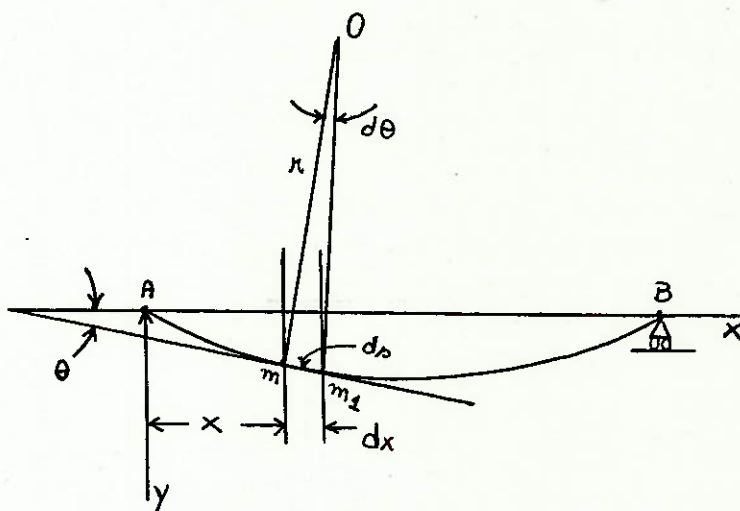
Tendo o comportamento da estrutura para cargas unitárias/nos padrões acima , uma carga de valor diferente da unidade/terá suas reações perfeitamente delineadas.

II - INTRODUÇÃO TEÓRICA

DEFORMAÇÃO DAS VIGAS CARREGADAS TRANSVERSALMENTE

EQUAÇÃO DIFERENCIAL DA LINHA ELÁSTICA :

No projeto de uma viga o engenheiro interessa-se, em geral, não somente pelas tensões produzidas pelas cargas que atuam nessa viga, mas, também, pela deformação produzida por essas cargas. Em muitos casos, além disso, especifica-se que o afundamento máximo não deve exceder a determinada pequena parte do vão.



Representamos pela curva AmB da figura acima, a forma do eixo da viga depois da flexão. A flexão se processa no plano de simetria devido às forças transversais que agem nesse plano. Esta curva é chamada de LINHA ELÁSTICA; Para deduzirmos a equação diferencial desta linha, tomemos os eixos de coordenadas como estão representados na figura e admitamos que a curvatura da linha elástica em qualquer ponto dependa, somente, da grandeza do momento fletor M naquele ponto. Neste caso, a relação entre a curvatura e o momento será a mesma do caso flexão pura (ver equação 1) e obteremos:

equação (1) :

$$\int \frac{E}{r} y^2 dA = \frac{EI_z}{r} = M \quad \text{ou} \quad \frac{1}{r} = \frac{M}{EI_z}$$

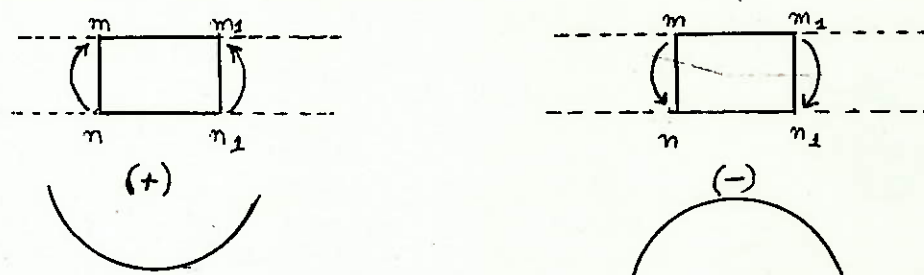
$$I_z = \int y^2 dA$$

obteremos : $\frac{1}{r} = \frac{M}{EI_z}$ (a)

Para deduzirmos uma expressão que nos dê a relação entre a curvatura e a forma da linha , devemos considerar dois pontos/ adjacentes m e m_1 na linha elástica , distantes um do outro de ds . Se o ângulo que a tangente em m faz com o eixo dos x fôr chamado de θ , o ângulo entre as normais à curva em m e m_1 se- rã $d\theta$. O ponto de interseção O destas normais dá o centro de/ curvatura e define o comprimento r do raio de curvatura. Então

$$ds = r d\theta \quad \text{e} \quad \frac{1}{r} = \left| \frac{d\theta}{ds} \right| \quad (b)$$

as barras indicando que consideramos aqui sòmente o valor nú - merico da curvatura. Com relação ao sinal devemos notar que o/ momento fletor é considerado positivo na equação (a) se produz concavidade virada para cima



Portanto , a curvatura será positiva quando o centro da curva, tura estiver acima da curva na da folha anterior. Entretanto , é fácil de ver que , para essa curvatura , o ângulo θ decresce á proporção que o ponto m se move ao longo da curva de A para/ B . Portanto a um aumento positivo ds correspondente um nega- tivo $d\theta$. Assim , para têmos o sinal conveniente , a equação / (b) deve ser escrita sob a forma

$$\frac{1}{r} = - \frac{d\theta}{ds} \quad (c)$$

Nas aplicações práticas só são permitidos afundamentos muito / pequenos das vigas e as linhas elásticas serão muito achata - das. Nesses casos podemos admitir , com precisão suficiente, / que

$$ds \approx dx \quad \text{e} \quad \theta \approx \text{tg } \theta = \frac{dy}{dx} \quad (d)$$

Levando êsses valores aproximados de ds e θ na equação (c) , obteremos :

$$\frac{1}{r} = - \frac{d^2y}{dx^2} \quad (e)$$

A equação (a) torna-se pois

$$EI_z \frac{d^2 y}{dx^2} = - M \quad (2)$$

Esta é a equação diferencial da linha elástica que deve ser integrada em cada caso particular, a fim de acharmos as deformações das vigas.

Devemos notar que o sinal na equação (2) depende da direção dos eixos coordenados. Por exemplo, se tomarmos os y positivos para cima é necessário que ponhamos

$$\theta \approx - \frac{dy}{dx}$$

no lugar da equação (d); e obteremos mais em vez de menos, no segundo membro da equação (2).

No caso de barras muito delgadas, onde a deformação ρ de ser grande, não é admissível que se unem as simplificações/ (d); e deve-se recorrer à expressão exata.

$$\theta = \arctg \left(- \frac{dy}{dx} \right)$$

Então:

$$\frac{1}{r} = - \frac{d\theta}{ds} = - \frac{d \arctg \left(\frac{dy}{dx} \right)}{dx} \frac{dx}{ds} = - \frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}} \quad (f)$$

Comparando este resultado com a equação (e), pode-se concluir/ que são equivalentes desde que se admita a quantidade $(dy/dx)^2$ no denominador da fórmula exata (f), pequena em presença da / unidade, podendo, portanto, ser desprezada.

Diferenciando a equação (2) em relação a x e aplicando as/ equações abaixo

$$dM = V dx \quad \frac{dM}{dx} = V$$

$$dV = - q dx \quad \frac{dV}{dx} = - q$$

obteremos:

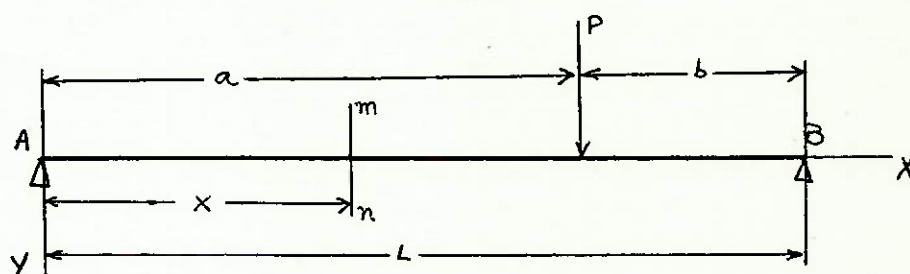
$$EI_z \frac{d^3 y}{dx^3} = -V$$

$$EI_z \frac{d^4 y}{dx^4} = q$$

A última equação é usada, algumas vezes, quando se considera / a deformação de vigas com carga distribuída.

DEFORMAÇÃO DE UMA VIGA SIMPLEMENTE APOIADA SOLICITADA POR UMA CARGA CONCENTRADA

Neste caso há duas expressões diferentes para o momento fletor, correspondentes às duas partes da viga, figura abaixo. A equação (1) da linha elástica deve, portanto, ser escrita para cada parte. Deste modo obtemos



$$EI_z \frac{d^2 y}{dx^2} = - \frac{Pb}{L} x \quad \text{para } x \leq a$$

$$EI_z \frac{d^2 y}{dx^2} = - \frac{Pb}{L} x + P(x-a) \quad \text{para } x \geq a$$

Integrando estas equações obtemos

$$\left. \begin{aligned}
 &EI_z \frac{dy}{dx} = - \frac{Pbx^2}{2L} + C \quad \text{para } x \leq a \\
 &EI_z \frac{dy}{dx} = - \frac{Pbx^2}{2L} + \frac{P(x-a)^2}{2} + C_1 \quad \text{para } x \geq a
 \end{aligned} \right\} \quad (a)$$

Como os dois ramos da linha elástica devem ter uma tangente comum no ponto de aplicação da carga P, as expressões acima (a) / para o deslocamento angular devem ser iguais para $x = a$.

Dáí concluímos que as constantes de integração são iguais, isto é, $C = C_1$. Efetuando a segunda integração e substituindo C_1 por C obtemos

$$EI_z y = - \frac{Pbx^3}{6L} + Cx + C_2 \quad \text{para } x \leq a \quad (b)$$

e

$$EI_z y = - \frac{Pbx^3}{6L} + \frac{P(x-a)^3}{6} + Cx + C_3 \quad \text{para } x \geq a$$

Como os dois ramos de linha elástica têm um afundamento comum / no ponto de aplicação da carga, as expressões (b) devem ser / idênticas para $x=a$. Daí segue-se que $C_2 = C_3$. Finalmente, necessitamos determinar somente as duas constantes C e C_2 para cuja determinação temos duas condições que são, a de serem os / afundamentos em cada uma das extremidades da viga, iguais a ze ro, Fazendo $x=0$ e $y = 0$ na primeira das expressões (b), acharemos

$$C_2 = C_3 = 0 \quad (c)$$

Fazendo $y = 0$ e $x=L$ na segunda das expressões (b) obteremos

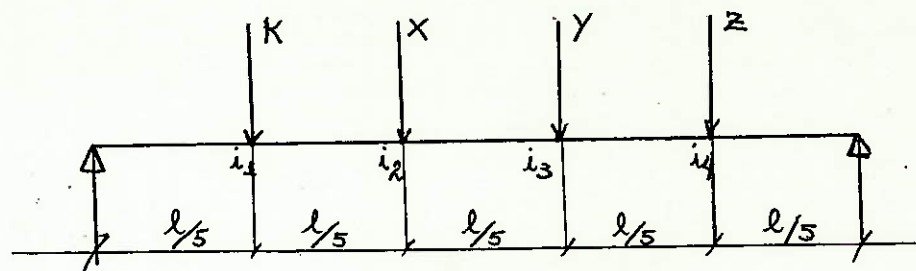
$$C = \frac{PbL}{6} - \frac{Pb^3}{6L} = \frac{Pb(L^2 - b^2)}{6L} \quad (d)$$

Levando os valores (c) e (d) das constantes nas equações (b) da linha elástica, obteremos

$$EI_z y = \frac{Pbx}{6L} (L^2 - b^2 - x^2) \quad \text{para } x \leq a$$

e

$$EI_z y = \frac{Pbx}{6L} (L^2 - b^2 - x^2) + P \frac{(x-a)^3}{6} \quad \text{para } x \geq a$$

III : MODELO TEÓRICO DAS VIGAS

K, X, Y, Z, esforços quaisquer aplicados aos nós i_1, i_2, i_3, i_4 - nós espaçados de $L/5$.

Utilizando as equações do item II temos :

FLECHA em i_1 :

devido a K

$$x = a = \frac{L}{5}$$

$$b = \frac{4L}{5}$$

$$y_K = \frac{PbL}{6LEI5} \left(L^2 - \frac{16}{25} L^2 - \frac{L^2}{25} \right)$$

$$y_K = \frac{4PbL^2}{375 EI}$$

$$y_K = \frac{4 K \frac{4L}{5} L^2}{5 \cdot 375 EI}$$

$$y_K = \frac{16 K L^3}{1875 EI} \quad (1)$$

devido a X

$$x = \frac{L}{5}$$

$$a = \frac{2L}{5}$$

$$b = \frac{3L}{5}$$

$$y_X = \frac{1}{EI} \cdot \frac{X \cdot 3L \cdot L}{6 \cdot 5L \cdot 5} \left(L^2 - \frac{9L^2}{25} - \frac{L^2}{25} \right)$$

$$y_X = \frac{3 L^3 X}{250 EI} \quad (2)$$

devido a Y

$$x = \frac{L}{5}$$

$$a = \frac{3L}{5}$$

$$b = \frac{2L}{5}$$

$$y_Y = \frac{1}{EI} \frac{Y \cdot 2L \cdot L}{6L \cdot 5 \cdot 5} \left(L^2 - \frac{4L^2}{25} - \frac{L^2}{25} \right)$$

$$y_Y = \frac{YL^3}{EI \cdot 75} \left(\frac{25 - 4 - 1}{25} \right)$$

$$y_Y = \frac{4L^3}{375} \frac{Y}{EI} \quad (3)$$

devido a Z

$$x = \frac{L}{5} \quad a = \frac{4L}{5} \quad b = \frac{L}{5}$$

$$y_Z = \frac{1}{EI} \frac{Z \cdot L \cdot L}{6L \cdot 5 \cdot 5} \left(L^2 - \frac{L^2}{25} - \frac{L^2}{25} \right)$$

$$y_Z = \frac{1}{EI} \frac{ZL^3}{150} \left(\frac{25 - 1 - 1}{25} \right)$$

$$y_Z = \frac{1}{EI} \frac{23 L^3}{3750} Z \quad (4)$$

Somando-se as expressões: (1), (2), (3), e (4), temos a flecha total na nó i_1 , devido aos esforços X, K, Y, Z:

$$y_{i_1} = \frac{L^3}{EI} \left[\frac{16}{1875} K + \frac{3}{250} X + \frac{4}{375} Y + \frac{23}{3750} Z \right]$$

De maneira análoga, calculamos as flechas em i_2 , i_3 , i_4 .

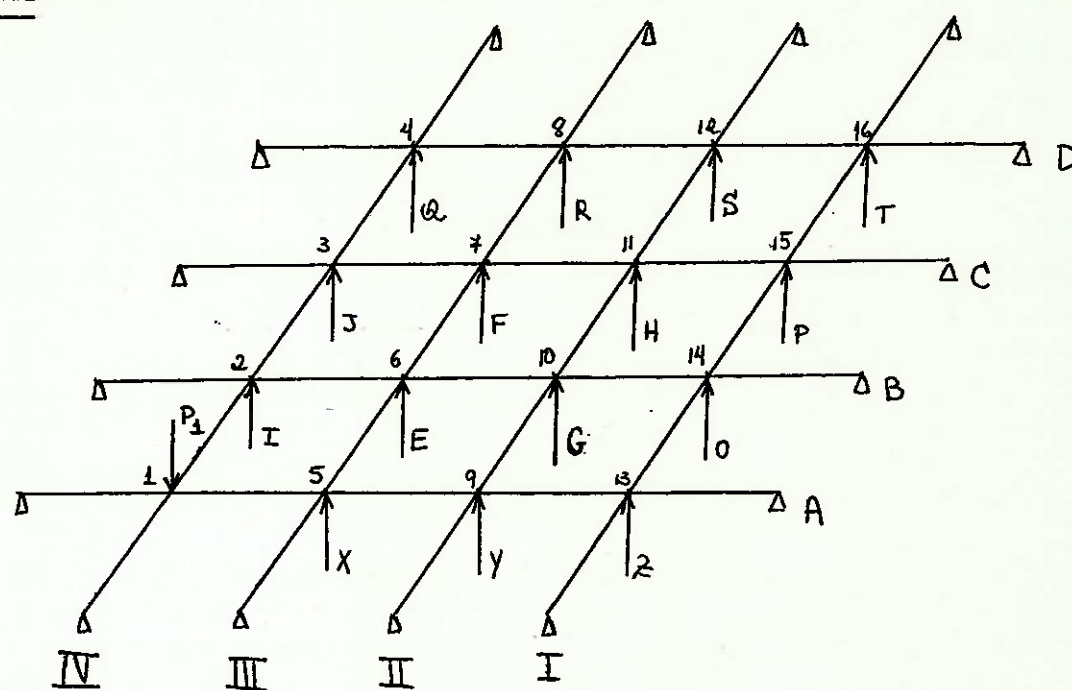
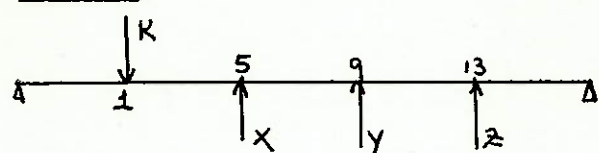
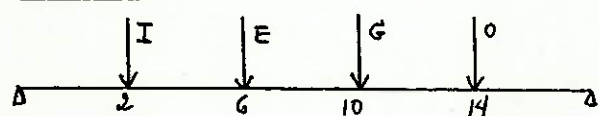
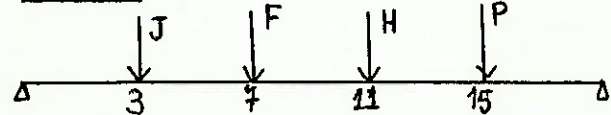
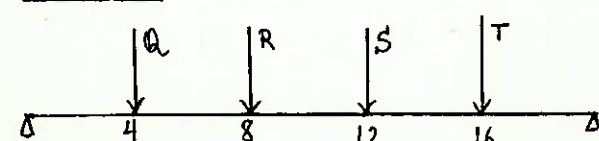
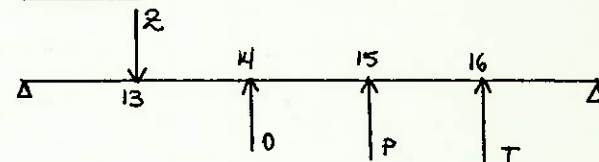
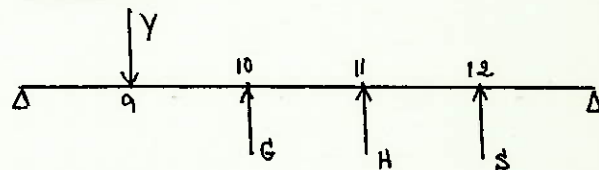
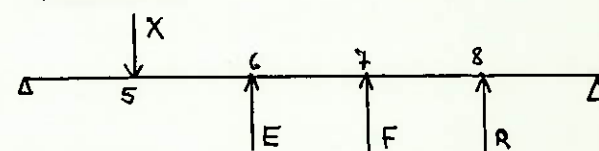
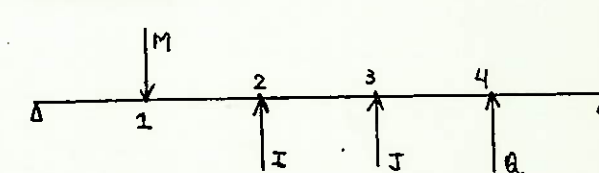
$$y_{i_2} = \frac{L^3}{EI} \left[\frac{3}{250} K + \frac{36}{1875} X + \frac{34}{1875} Y + \frac{4}{375} Z \right]$$

$$y_{i_3} = \frac{L^3}{EI} \left[\frac{4}{375} K + \frac{68}{3750} X + \frac{12}{625} Y + \frac{3}{250} Z \right]$$

$$y_{i_4} = \frac{L^3}{EI} \left[\frac{23}{3750} K + \frac{4}{375} X + \frac{9}{750} Y + \frac{16}{1875} Z \right]$$

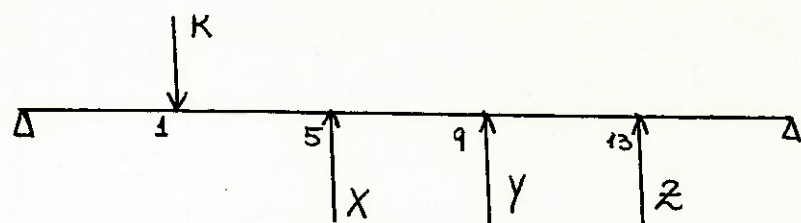
Nos cálculos subsequentes, utilizaremos estas expressões para dedução dos deslocamentos nos diversos nós.

IV - CÁLCULOS DOS SISTEMAS

CASO AVIGA AVIGA BVIGA CVIGA DVIGA IVIGA IIVIGA IIIVIGA IV

onde $K + M = 1$

VIGA A :



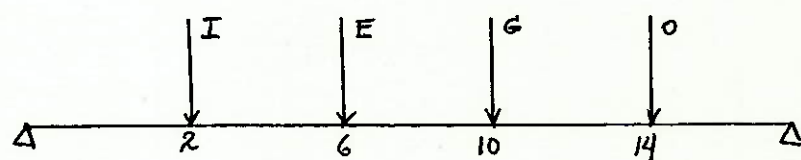
$$y_1 = \frac{L^3}{EI} \left[\frac{16}{1875} K - \frac{3}{250} X - \frac{4}{375} Y - \frac{23}{3750} Z \right]$$

$$y_5 = \frac{L^3}{EI} \left[\frac{3}{250} K - \frac{36}{1875} X - \frac{34}{1875} Y - \frac{4}{375} Z \right]$$

$$y_9 = \frac{L^3}{EI} \left[\frac{4}{375} K - \frac{68}{3750} X - \frac{12}{625} Y - \frac{3}{250} Z \right]$$

$$y_{13} = \frac{L^3}{EI} \left[\frac{23}{3750} K - \frac{4}{375} X - \frac{9}{750} Y - \frac{16}{1875} Z \right]$$

VIGA B :

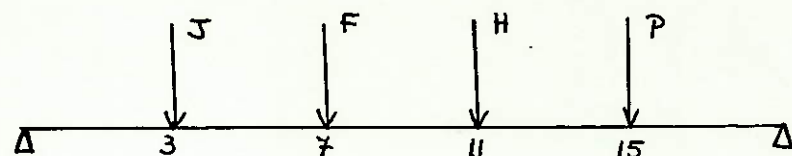


$$y_2 = \frac{L^3}{EI} \left[\frac{16}{1875} I + \frac{3}{250} E + \frac{4}{375} G + \frac{23}{3750} O \right]$$

$$y_6 = \frac{L^3}{EI} \left[\frac{3}{250} I + \frac{36}{1875} E + \frac{34}{1875} G + \frac{4}{375} O \right]$$

$$y_{10} = \frac{L^3}{EI} \left[\frac{4}{375} I + \frac{68}{3750} E + \frac{12}{625} G + \frac{3}{250} O \right]$$

$$y_{14} = \frac{L^3}{EI} \left[\frac{23}{3750} I + \frac{4}{375} E + \frac{9}{750} G + \frac{16}{1875} O \right]$$

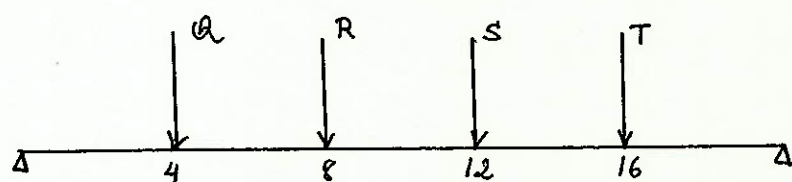
VIGA C:

$$y_3 = \frac{L^3}{EI} \left[\frac{16}{1875} J + \frac{3}{250} F + \frac{4}{375} H + \frac{23}{3750} P \right]$$

$$y_7 = \frac{L^3}{EI} \left[\frac{3}{250} J + \frac{36}{1875} F + \frac{34}{1875} H + \frac{4}{375} P \right]$$

$$y_{11} = \frac{L^3}{EI} \left[\frac{4}{375} J + \frac{68}{3750} F + \frac{12}{625} H + \frac{3}{250} P \right]$$

$$y_{15} = \frac{L^3}{EI} \left[\frac{23}{3750} J + \frac{4}{375} F + \frac{9}{750} H + \frac{16}{1875} P \right]$$

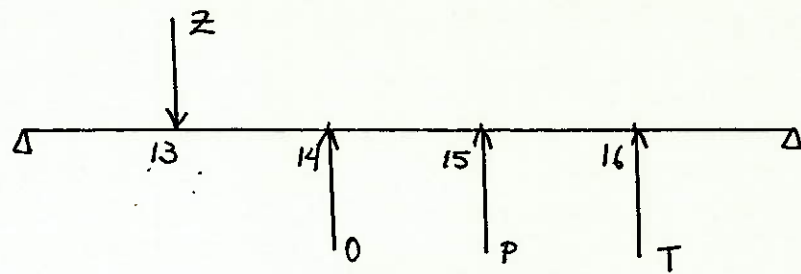
VIGA D :

$$\bar{y}_4 = \frac{L^3}{EI} \left[\frac{16}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T \right]$$

$$y_8 = \frac{L^3}{EI} \left[\frac{3}{250} Q + \frac{36}{1875} R + \frac{34}{1875} S + \frac{4}{375} T \right]$$

$$y_{12} = \frac{L^3}{EI} \left[\frac{4}{375} Q + \frac{68}{3750} R + \frac{12}{625} S + \frac{3}{250} T \right]$$

$$y_{16} = \frac{L^3}{EI} \left[\frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{16}{1875} T \right]$$

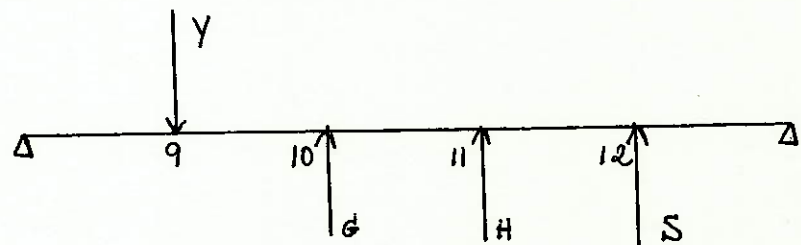
VIGA I :

$$y_{13} = \frac{L^3}{EI} \left[\frac{16}{1875} Z - \frac{3}{250} 0 - \frac{4}{375} P - \frac{23}{3750} T \right]$$

$$y_{14} = \frac{L^3}{EI} \left[\frac{3}{250} Z - \frac{36}{1875} 0 - \frac{34}{1875} P - \frac{4}{375} T \right]$$

$$y_{15} = \frac{L^3}{EI} \left[\frac{4}{375} Z - \frac{68}{3750} 0 - \frac{12}{625} P - \frac{3}{250} T \right]$$

$$y_{16} = \frac{L^3}{EI} \left[\frac{23}{3750} Z - \frac{4}{375} 0 - \frac{9}{750} P - \frac{16}{1875} T \right]$$

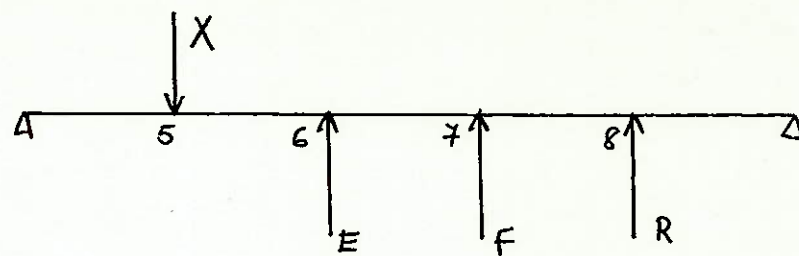
VIGA II :

$$y_9 = \frac{L^3}{EI} \left[\frac{16}{1875} Y - \frac{3}{250} G - \frac{4}{375} H - \frac{23}{3750} S \right]$$

$$y_{10} = \frac{L^3}{EI} \left[\frac{3}{250} Y - \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S \right]$$

$$y_{11} = \frac{L^3}{EI} \left[\frac{4}{375} Y - \frac{68}{3750} G - \frac{12}{625} H - \frac{3}{250} S \right]$$

$$y_{12} = \frac{L^3}{EI} \left[\frac{23}{3750} Y - \frac{4}{375} G - \frac{9}{750} H - \frac{16}{1875} S \right]$$

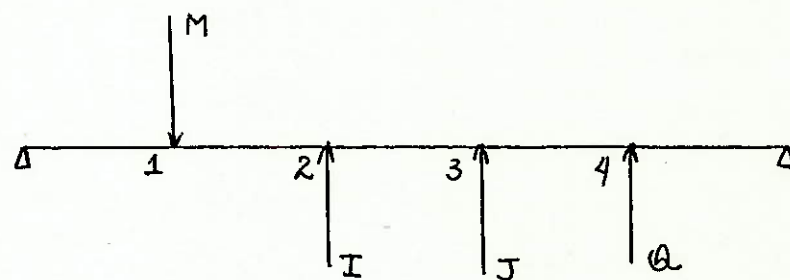
VIGA III:

$$y_5 = \frac{L^3}{EI} \left[\frac{16}{1875} X - \frac{3}{250} E - \frac{4}{375} F - \frac{23}{3750} R \right]$$

$$y_6 = \frac{L^3}{EI} \left[\frac{3}{250} X - \frac{36}{1875} E - \frac{34}{1875} F - \frac{4}{375} R \right]$$

$$y_7 = \frac{L^3}{EI} \left[\frac{4}{375} X - \frac{68}{3750} E - \frac{12}{625} F - \frac{3}{250} R \right]$$

$$y_8 = \frac{L^3}{EI} \left[\frac{23}{3750} X - \frac{4}{375} E - \frac{9}{750} F - \frac{16}{1875} R \right]$$

VIGA IV :

$$y_1 = \frac{L^3}{EI} \left[\frac{16}{1875} M - \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q \right]$$

$$y_2 = \frac{L^3}{EI} \left[\frac{3}{250} M - \frac{36}{1875} I - \frac{34}{1875} J - \frac{4}{375} Q \right]$$

$$y_3 = \frac{L^3}{EI} \left[\frac{4}{375} M - \frac{68}{3750} I - \frac{12}{625} J - \frac{3}{250} Q \right]$$

$$y_4 = \frac{L^3}{EI} \left[\frac{23}{3750} M - \frac{4}{375} I - \frac{9}{750} J - \frac{16}{1875} Q \right]$$

IGUALANDO-SE AS FLECHAS RESPECTIVAS , TEMOS -

Nó 1 :

$$\frac{L^3}{EI} \left[\frac{16}{1875} K - \frac{3X}{250} - \frac{4Y}{375} - \frac{23Z}{3750} \right] = \frac{L^3}{EI} \left[\frac{16}{1875} M - \frac{3I}{250} - \frac{4J}{375} - \frac{23Q}{3750} \right]$$

$$\frac{3}{250} I + \frac{4}{375} J + \frac{16}{1875} K - \frac{16}{1875} M + \frac{23}{3750} Q - \frac{3}{250} X - \frac{4}{375} Y - \frac{23}{3750} Z = 0$$

$$\text{como } K + M = 1 \implies M = 1 - K$$

e substituindo , temos ::

$$\frac{3}{250} I + \frac{4}{375} J + \frac{32}{1875} K + \frac{23}{3750} Q - \frac{3}{250} X - \frac{4}{375} Y - \frac{23}{3750} Z = \frac{16}{1875}$$

Nó 2 :

$$\frac{16}{1875} I + \frac{3}{250} E + \frac{4}{375} G + \frac{23}{3750} O = \frac{3}{250} M - \frac{36}{1875} I - \frac{34}{1875} J - \frac{4}{375} Q$$

$$\frac{52}{1875} I + \frac{34}{1875} J + \frac{4}{375} Q - \frac{3}{250} M + \frac{3}{250} E + \frac{4}{375} G + \frac{23}{3750} O = 0$$

substituindo M por (1 - K) temos :

$$\frac{3}{250} E + \frac{4}{375} G + \frac{52}{1875} I + \frac{34}{1875} J + \frac{3}{250} K + \frac{23}{3750} O + \frac{4}{375} Q = \frac{3}{250}$$

Nó 3 :

$$\frac{16}{1375} J + \frac{3}{250} F + \frac{4}{375} H + \frac{23}{3750} P = \frac{4}{375} M - \frac{68}{3750} I - \frac{12}{625} J - \frac{3}{250} Q$$

$$M = 1 - K \quad \therefore$$

$$\frac{3}{250} F + \frac{4}{375} H + \frac{68}{3750} I + \frac{52}{1875} J + \frac{4}{375} K + \frac{23}{3750} P + \frac{3}{250} Q = \frac{4}{375}$$

Nó 4 :

$$\frac{16}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T = \frac{23}{3750} M - \frac{4}{375} I - \frac{9}{750} J - \frac{16}{1875} Q$$

$$M = 1 - K \quad \therefore$$

$$\frac{4}{375} I + \frac{9}{750} J + \frac{23}{3750} K + \frac{32}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T = \frac{23}{3750}$$

Nó 5 :

$$\frac{16}{1875} X - \frac{3}{250} E - \frac{4}{375} F - \frac{23}{3750} R = \frac{3}{250} K - \frac{36}{1875} X - \frac{34}{1875} Y - \frac{4}{375} Z$$

$$- \frac{3}{250} E - \frac{4}{375} F - \frac{3}{250} K - \frac{23}{3750} R + \frac{52}{1875} X + \frac{34}{1875} Y + \frac{4}{375} Z = 0$$

Nó 6 -:

$$\frac{3}{250} I + \frac{36}{1875} E + \frac{34}{1875} G + \frac{4}{375} O = \frac{3}{250} X - \frac{36}{1875} E - \frac{34}{1875} F - \frac{4}{375} R$$

$$\frac{72}{1875} E + \frac{34}{1875} F + \frac{34}{1875} G + \frac{3}{250} I + \frac{4}{375} O + \frac{4}{375} R - \frac{3}{250} X = 0$$

Nó 7 :

$$\frac{3}{250} J + \frac{36}{1875} F + \frac{34}{1875} H + \frac{4}{375} P = \frac{4}{375} X - \frac{68}{3750} E - \frac{12}{625} F - \frac{3}{250} R$$

$$\frac{68}{3750} E + \frac{72}{1875} F + \frac{34}{1875} H + \frac{3}{250} J + \frac{4}{375} P + \frac{3}{250} R - \frac{4}{375} X = 0$$

Nó 8 :

$$\frac{23}{3750} X - \frac{4}{375} E - \frac{9}{750} F - \frac{16}{1875} R = \frac{3}{250} Q + \frac{36}{1875} R + \frac{34}{1875} S + \frac{4}{375} T$$

$$\frac{4}{375} E + \frac{9}{750} F + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T - \frac{23}{3750} X = 0$$

Nó 9 :

$$\frac{4}{375} K - \frac{68}{3750} X - \frac{12}{625} Y - \frac{3}{250} Z = \frac{16}{1875} Y - \frac{3}{250} G - \frac{4}{375} H - \frac{23}{3750} S$$

$$\frac{3}{250} G + \frac{4}{375} H + \frac{4}{375} K + \frac{23}{3750} S - \frac{68}{3750} X - \frac{52}{1875} Y - \frac{3}{250} Z = 0$$

Nó 10 :

$$\frac{4}{375} I + \frac{68}{3750} E + \frac{12}{625} G + \frac{3}{250} O = \frac{3}{250} Y - \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S$$

$$\frac{68}{3750} E + \frac{72}{1875} G + \frac{34}{1875} H + \frac{4}{375} I + \frac{3}{250} O + \frac{4}{375} S - \frac{3}{250} Y = 0$$

Nó 11 :

$$\frac{4}{375}J + \frac{68}{3750}F + \frac{12}{625}H + \frac{3}{250}P = \frac{4}{375}Y - \frac{68}{3750}G - \frac{12}{625}H - \frac{3}{250}S$$

$$\frac{68}{3750}F + \frac{68}{3750}G + \frac{24}{625}H + \frac{4}{375}J + \frac{3}{250}P + \frac{3}{250}S - \frac{4}{375}Y = 0$$

Nó 12 :

$$\frac{23}{3750}Y - \frac{4}{375}G - \frac{9}{750}H - \frac{16}{1875}S = \frac{4}{375}Q + \frac{68}{3750}R + \frac{12}{625}S + \frac{3}{250}T$$

$$\frac{4}{375}G + \frac{9}{750}H + \frac{4}{375}Q + \frac{68}{3750}R + \frac{52}{1875}S + \frac{3}{250}T - \frac{23}{3750}Y = 0$$

Nó 13 :

$$\frac{23}{3750}K - \frac{4}{375}X - \frac{9}{750}Y - \frac{16}{1875}Z = \frac{16}{1875}Z - \frac{3}{250}O - \frac{4}{375}P - \frac{23}{3750}T$$

$$\frac{23}{3750}K + \frac{3}{250}O + \frac{4}{375}P + \frac{23}{3750}T - \frac{4}{375}X - \frac{9}{750}Y - \frac{32}{1875}Z = 0$$

Nó 14 :

$$\frac{23}{3750}I + \frac{4}{375}E + \frac{9}{750}G + \frac{16}{1875}O = \frac{3}{250}Z - \frac{36}{1875}O - \frac{34}{1875}P - \frac{4}{375}T$$

$$\frac{4}{375}E + \frac{9}{750}G + \frac{23}{3750}I + \frac{52}{1875}O + \frac{34}{1875}P + \frac{4}{375}T - \frac{3}{250}Z = 0$$

Nó 15 :

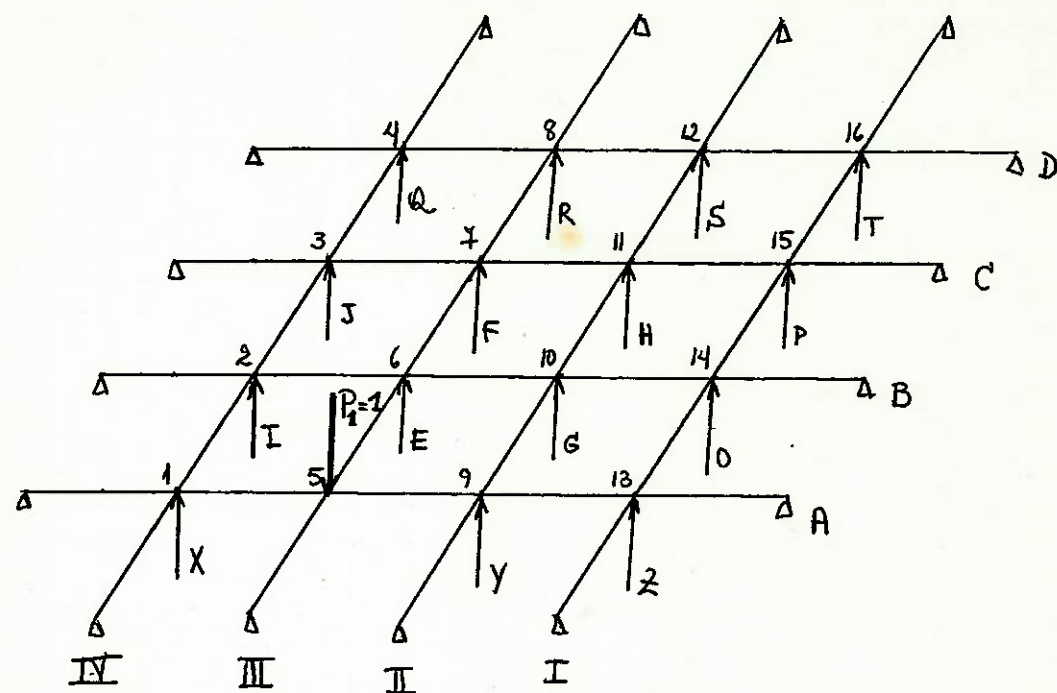
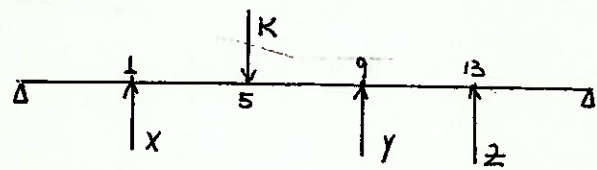
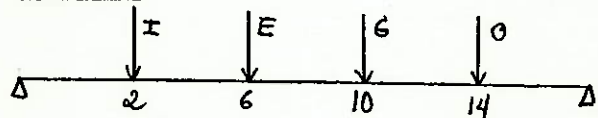
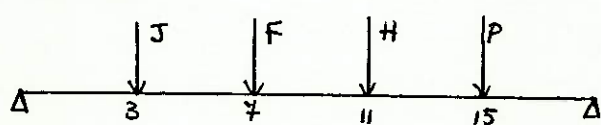
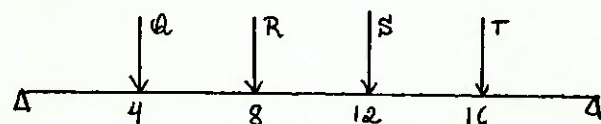
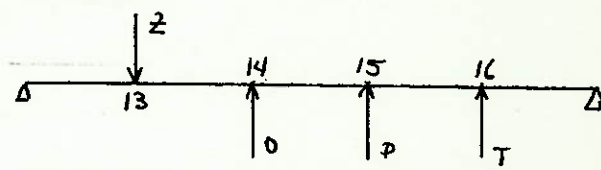
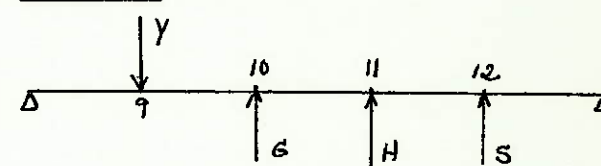
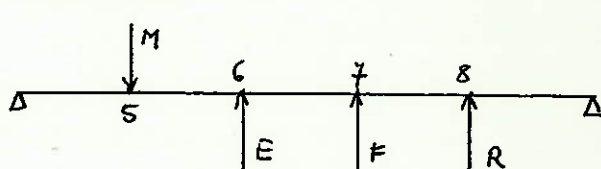
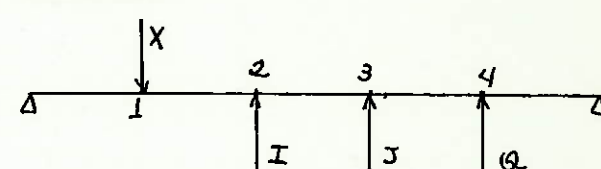
$$\frac{23}{3750}J + \frac{4}{375}F + \frac{9}{750}H + \frac{16}{1875}P = \frac{4}{375}Z - \frac{68}{3750}O - \frac{12}{625}P - \frac{3}{250}T$$

$$\frac{4}{375} F + \frac{9}{750} H + \frac{23}{3750} J + \frac{68}{3750} O + \frac{52}{1875} P + \frac{3}{250} T - \frac{4}{375} Z = 0$$

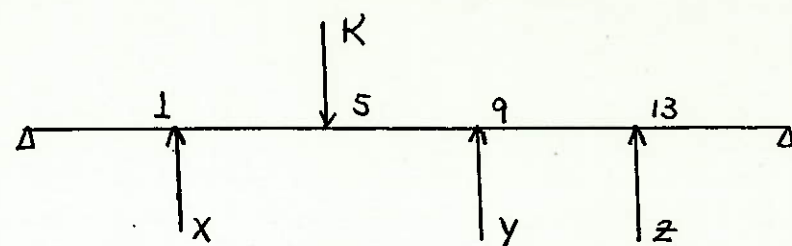
Nº 16 :

$$\frac{23}{3750} Z - \frac{4}{375} O - \frac{9}{750} P - \frac{16}{1875} T = \frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{16}{1875} T$$

$$\frac{4}{375} O + \frac{9}{750} P + \frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{32}{1875} T - \frac{23}{3750} Z = 0$$

CASO B.VIGA AVIGA BVIGA CVIGA DVIGA IVIGA IIVIGA IIIVIGA IV

$$K + M = 1$$

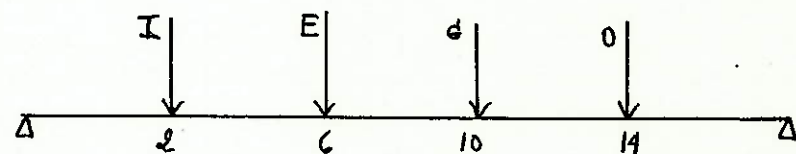
VIGA A :

$$y_1 = \frac{L^3}{EI} \left[-\frac{16}{1875} X + \frac{3}{250} K - \frac{4}{375} Y - \frac{23}{3750} Z \right]$$

$$y_5 = \frac{L^3}{EI} \left[-\frac{3}{250} X + \frac{36}{1875} K - \frac{34}{1875} Y - \frac{4}{375} Z \right]$$

$$y_9 = \frac{L^3}{EI} \left[-\frac{4}{375} X + \frac{68}{3750} K - \frac{12}{625} Y - \frac{3}{250} Z \right]$$

$$y_{13} = \frac{L^3}{EI} \left[-\frac{23}{3750} X + \frac{4}{375} K - \frac{9}{750} Y - \frac{16}{1875} Z \right]$$

VIGA B :

$$y_2 = \frac{L^3}{EI} \left[\frac{16}{1875} I + \frac{3}{250} E + \frac{4}{375} G + \frac{23}{3750} 0 \right]$$

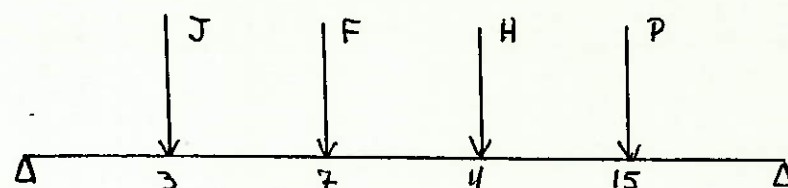
$$y_6 = \frac{L^3}{EI} \left[\frac{3}{250} I + \frac{36}{1875} E + \frac{34}{1875} G + \frac{4}{375} 0 \right]$$

$$y_{10} = \frac{L^3}{EI} \left[\frac{4}{375} I + \frac{68}{3750} E + \frac{12}{625} G + \frac{3}{250} 0 \right]$$

$$y_{14} = \frac{L^3}{EI} \left[\frac{23}{3750} I + \frac{4}{375} E + \frac{9}{750} G + \frac{16}{1875} 0 \right]$$

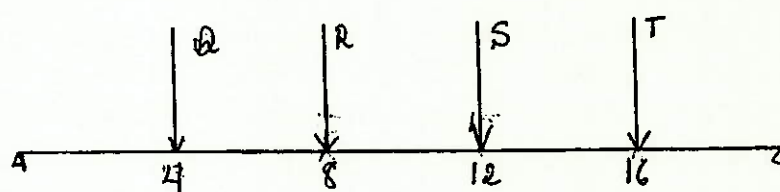
(igual ao caso A)

VIGA C :



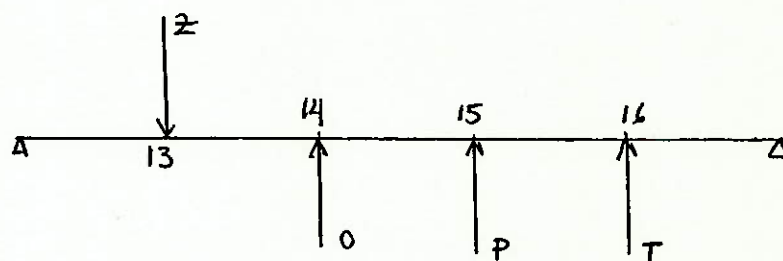
igual ao caso A .

VIGA D :



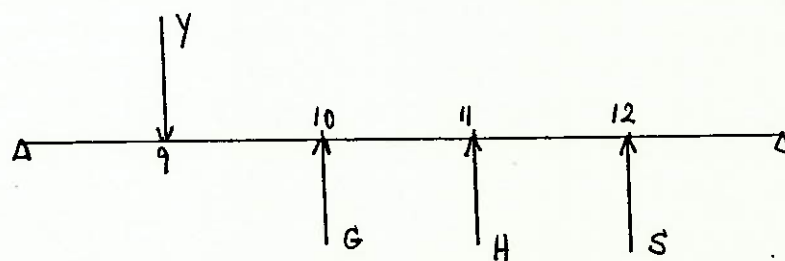
igual ao caso A.

VIGA I :



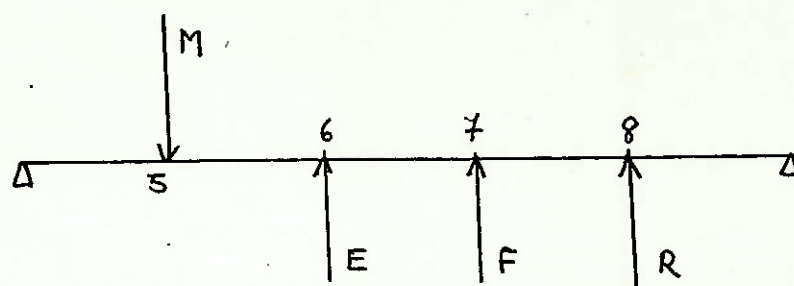
igual ao caso A.

VIGA II :



igual ao caso A .

VIGA III :



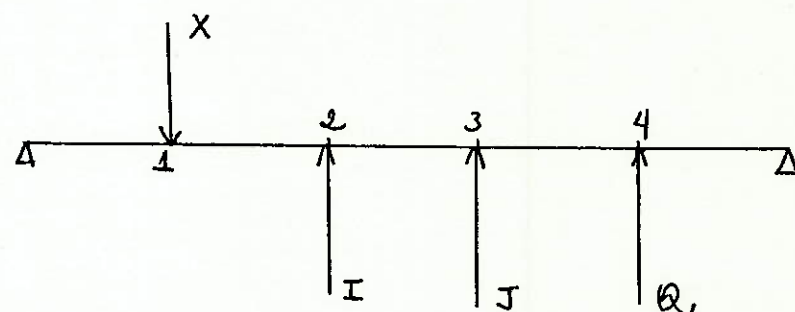
$$y_5 = \frac{L^3}{EI} \left[\frac{16}{1875} M - \frac{3}{250} E - \frac{4}{375} F - \frac{23}{3750} R \right]$$

$$y_6 = \frac{L^3}{EI} \left[\frac{3}{250} M - \frac{36}{1875} E - \frac{34}{1875} F - \frac{4}{375} R \right]$$

$$y_7 = \frac{L^3}{EI} \left[\frac{4}{375} M - \frac{68}{3750} E - \frac{12}{625} F - \frac{3}{250} R \right]$$

$$y_8 = \frac{L^3}{EI} \left[\frac{23}{3750} M - \frac{4}{375} E - \frac{9}{750} F - \frac{16}{1875} R \right]$$

VIGA IV :



$$y_1 = \frac{L^3}{EI} \left[\frac{16}{1875} X - \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q \right]$$

$$y_2 = \frac{L^3}{EI} \left[\frac{3}{250} X - \frac{36}{1875} I - \frac{34}{1875} J - \frac{4}{375} Q \right]$$

$$y_3 = \frac{L^3}{EI} \left[\frac{4}{375} X - \frac{68}{3750} I - \frac{12}{625} J - \frac{3}{250} Q \right]$$

$$y_4 = \frac{L^3}{EI} \left[\frac{23}{3750} X - \frac{4}{375} I - \frac{9}{750} J - \frac{16}{1875} Q \right]$$

" " " "

IGUALANDO - SE AS FLECHAS RESPECTIVAS :

NÓ (1) :

$$\frac{16}{1875} X - \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q = - \frac{16}{1875} X + \frac{3}{250} K - \frac{4}{375} Y - \frac{23}{3750} Z$$

$$- \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q - \frac{3}{250} K + \frac{32}{1875} X + \frac{4}{375} Y + \frac{23}{3750} Z = 0$$

NÓ (2) :

$$\frac{3}{250} X - \frac{36}{1875} I - \frac{34}{1875} J - \frac{4}{375} Q = \frac{16}{1875} I + \frac{3}{250} E + \frac{4}{375} G + \frac{23}{3750} O$$

$$\frac{3}{250} E + \frac{4}{375} G + \frac{52}{1875} I + \frac{34}{1875} J + \frac{23}{3750} O + \frac{4}{375} Q - \frac{3}{250} X = 0$$

NÓ (3) :

$$\frac{4}{375} X - \frac{68}{3750} I - \frac{12}{625} J - \frac{3}{250} Q = \frac{16}{1875} J + \frac{3}{250} F + \frac{4}{375} H + \frac{23}{3750} P$$

$$\frac{3}{250} F + \frac{4}{375} H + \frac{68}{3750} I + \frac{52}{1875} J + \frac{23}{3750} P + \frac{3}{250} Q - \frac{4}{375} X = 0$$

Nº (4) :

$$\frac{23}{3750} X - \frac{4}{375} I - \frac{9}{750} J - \frac{16}{1875} Q = \frac{16}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T$$

$$\frac{4}{375} I + \frac{9}{750} J + \frac{32}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T - \frac{23}{3750} X = 0$$

Nº (5) :

$$-\frac{3}{250} X + \frac{36}{1875} K - \frac{34}{1875} Y - \frac{4}{375} Z = \frac{16}{1875} M - \frac{3}{250} E - \frac{4}{375} F - \frac{23}{3750} R$$

$$\frac{3}{250} E + \frac{4}{375} F + \frac{36}{1875} K - \frac{16}{1875} M + \frac{23}{3750} R - \frac{3}{250} X - \frac{34}{1875} Y - \frac{4}{375} Z = 0$$

Nº (6) :

$$\frac{3}{250} I + \frac{36}{1875} E + \frac{34}{1875} G + \frac{4}{375} O = \frac{3}{250} M - \frac{36}{1875} E - \frac{34}{1875} F - \frac{4}{375} R$$

$$\frac{72}{1875} E + \frac{34}{1875} F + \frac{34}{1875} G + \frac{3}{250} I - \frac{3}{250} M + \frac{4}{375} O + \frac{4}{375} R = 0$$

Nº (7) :

$$\frac{3}{250} J + \frac{36}{1875} F + \frac{34}{1875} H + \frac{4}{375} P = \frac{4}{375} M - \frac{68}{3750} E - \frac{12}{625} F - \frac{3}{250} R$$

$$\frac{68}{3750} E + \frac{72}{1875} F + \frac{34}{1875} H + \frac{3}{250} J - \frac{4}{375} M + \frac{4}{375} P + \frac{3}{250} R = 0$$

Nº (8) :

$$\frac{3}{250} Q + \frac{36}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = \frac{23}{3750} M - \frac{4}{375} E - \frac{9}{750} F - \frac{16}{1875} R$$

$$\frac{4}{375} E + \frac{9}{750} F - \frac{23}{3750} M + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = 0$$

Nº (9) :

$$- \frac{4}{375} X + \frac{68}{3750} K - \frac{12}{625} Y - \frac{3}{250} Z = \frac{16}{1875} Y - \frac{3}{250} G - \frac{4}{375} H - \frac{23}{3750} S$$

$$\frac{3}{250} G + \frac{4}{375} H + \frac{68}{3750} K + \frac{23}{3750} S - \frac{4}{375} X - \frac{52}{1875} Y - \frac{3}{250} Z = 0$$

Nº (10) :

$$\frac{4}{375} I + \frac{68}{3750} E + \frac{12}{625} G + \frac{3}{250} O = \frac{3}{250} Y - \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S$$

$$\frac{68}{3750} E + \frac{72}{1875} G + \frac{34}{1875} H + \frac{4}{375} I + \frac{3}{250} O + \frac{4}{375} S - \frac{3}{250} Y = 0$$

NÓ (11) :

$$\frac{68}{3750} F + \frac{68}{3750} G + \frac{24}{625} H + \frac{4}{375} J + \frac{3}{250} P + \frac{3}{250} S - \frac{4}{375} Y = 0$$

NÓ (12) :

$$\frac{4}{375} G + \frac{9}{750} H + \frac{4}{375} Q + \frac{68}{3750} R + \frac{52}{1875} S + \frac{3}{250} T - \frac{23}{3750} Y = 0$$

NÓ (13) :

$$-\frac{23}{3750} X + \frac{4}{375} K - \frac{9}{750} Y - \frac{16}{1875} Z = \frac{16}{1875} Z - \frac{3}{250} O - \frac{4}{375} P - \frac{23}{3750} T$$

$$\frac{4}{375} K + \frac{3}{250} O + \frac{4}{375} P + \frac{23}{3750} T - \frac{23}{3750} X - \frac{9}{750} Y - \frac{32}{1875} Z = 0$$

NÓ (14) :

$$\frac{4}{375} E + \frac{9}{750} G + \frac{23}{3750} I + \frac{52}{1875} O + \frac{34}{1875} P + \frac{4}{375} T - \frac{3}{250} Z = 0$$

Nº (15) :

$$\frac{4}{375} F + \frac{9}{750} H + \frac{23}{3750} J + \frac{68}{3750} O + \frac{52}{1875} P + \frac{3}{250} T - \frac{4}{375} Z = 0$$

Nº (16) :

$$\frac{4000}{375} + \frac{9}{750} P + \frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{32}{1875} T - \frac{23}{3750} Z = 0$$

$$e \quad \underline{K + M = 1} \quad \underline{\quad} \quad \underline{M = 1 - K} \quad e$$

substituindo nas equações respectivas , temos :

Nº (5) :

$$\frac{3}{250} E + \frac{4}{375} F + \frac{36}{1875} K + \frac{16}{1875} K + \frac{23}{3750} R - \frac{3}{250} X - \frac{34}{1875} Y - \frac{4}{375} Z - \frac{16}{1875} = 0$$

$$\frac{3}{250} E + \frac{4}{375} F + \frac{52}{1875} K + \frac{23}{3750} R - \frac{3}{250} X - \frac{34}{1875} Y - \frac{4}{375} Z = \frac{16}{1875}$$

Nº (6) :

$$\frac{72}{1875} E + \frac{34}{1875} F + \frac{34}{1875} G + \frac{3}{250} I - \frac{3}{250} + \frac{3}{250} K + \frac{4}{375} O + \frac{4}{375} R = 0$$

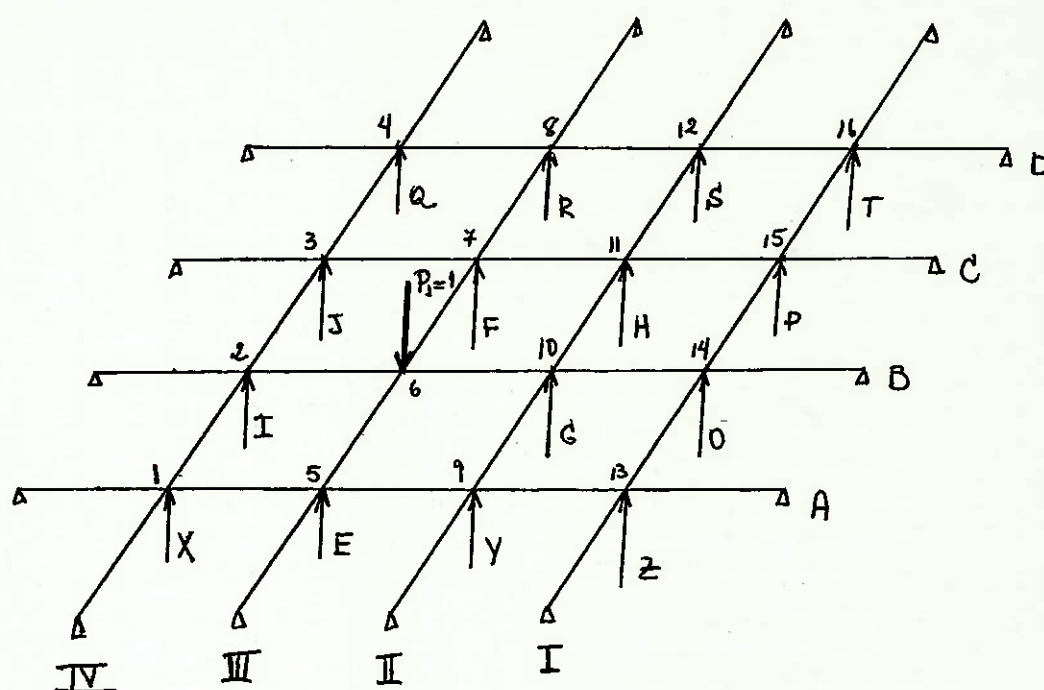
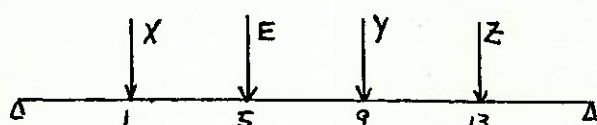
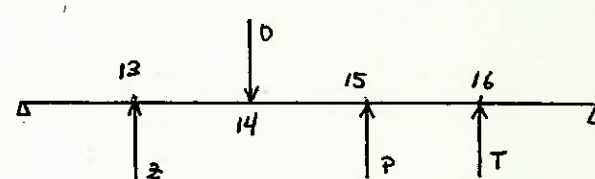
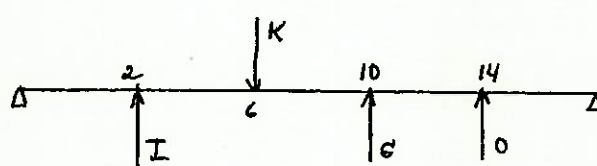
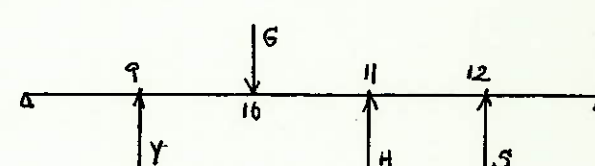
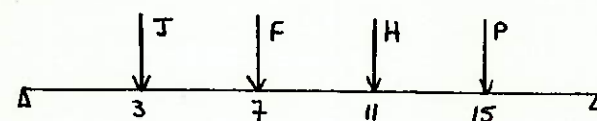
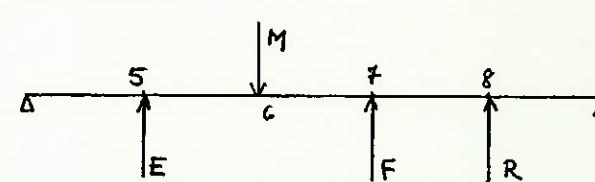
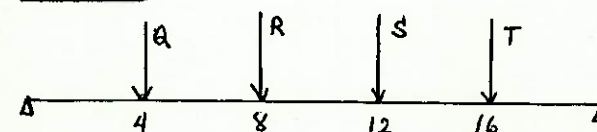
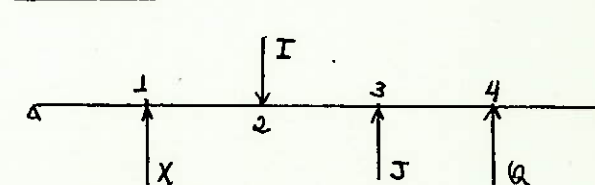
$$\frac{72}{1875} E + \frac{34}{1875} F + \frac{34}{1875} G + \frac{3}{250} I + \frac{3}{250} K + \frac{4}{375} O + \frac{4}{375} R = \frac{3}{250}$$

Nº (7) :

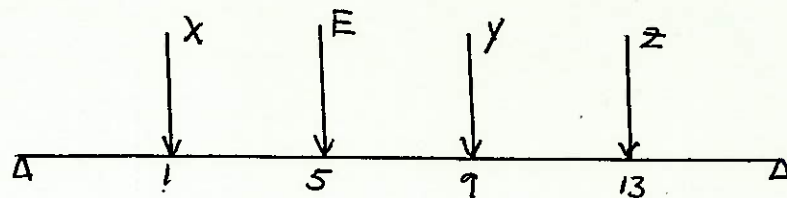
$$\frac{68}{3750} E + \frac{72}{1875} F + \frac{34}{1875} H + \frac{3}{250} J + \frac{4}{375} K + \frac{4}{375} P + \frac{3}{250} R = \frac{4}{375}$$

Nº (8) :

$$\frac{4}{375} E + \frac{9}{750} F + \frac{23}{3750} K + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = \frac{23}{3750}$$

CASO CVIGA AVIGA IVIGA BVIGA IIVIGA CVIGA IIIVIGA DVIGA IV

$$K + M = 1$$

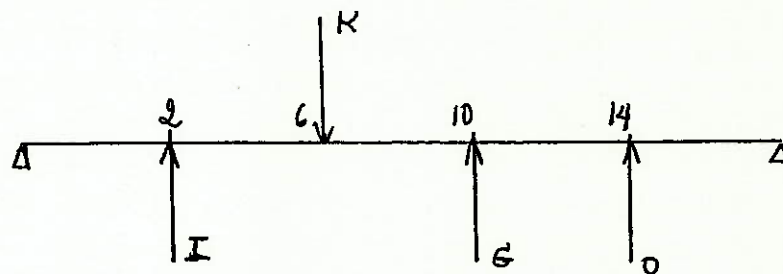
VIGA A

$$y_1 = \frac{L^3}{EI} \left[\frac{16}{1875} X + \frac{3}{250} E + \frac{4}{375} Y + \frac{23}{3750} Z \right]$$

$$y_5 = \frac{L^3}{EI} \left[\frac{3}{250} X + \frac{36}{1875} E + \frac{34}{1875} Y + \frac{4}{375} Z \right]$$

$$y_9 = \frac{L^3}{EI} \left[\frac{4}{375} X + \frac{68}{3750} E + \frac{12}{625} Y + \frac{3}{250} Z \right]$$

$$y_{13} = \frac{L^3}{EI} \left[\frac{23}{3750} X + \frac{4}{375} E + \frac{9}{750} Y + \frac{16}{1875} Z \right]$$

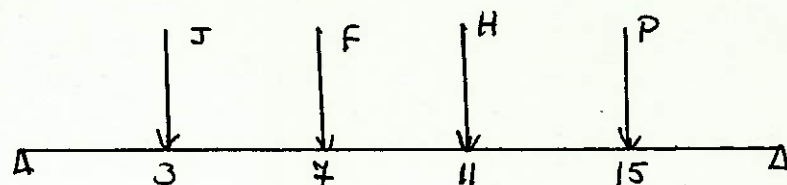
VIGA B

$$y_2 = \frac{L^3}{EI} \left[-\frac{16}{1875} I + \frac{3}{250} K - \frac{4}{375} G - \frac{23}{3750} O \right]$$

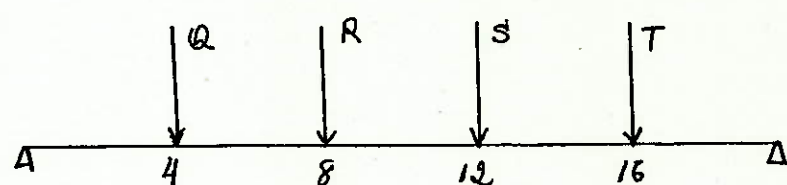
$$y_6 = \frac{L^3}{EI} \left[-\frac{3}{250} I + \frac{36}{1875} K - \frac{34}{1875} G - \frac{4}{375} O \right]$$

$$y_{10} = \frac{L^3}{EI} \left[-\frac{4}{375} I + \frac{68}{3750} K - \frac{12}{625} G - \frac{3}{250} O \right]$$

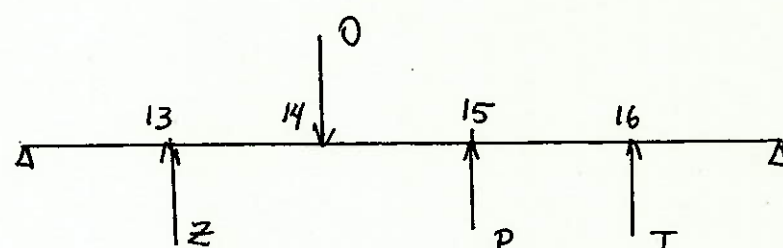
$$y_{14} = \frac{L^3}{EI} \left[-\frac{23}{3750} I + \frac{4}{375} K - \frac{9}{750} G - \frac{16}{1875} O \right]$$

VIGA C

igual aos casos anteriores.

VIGA D

igual aos casos anteriores.

VIGA I

$$y_{13} = \frac{L^3}{EI} \left[-\frac{16}{1875} Z + \frac{3}{250} 0 - \frac{4}{375} P - \frac{23}{3750} T \right]$$

$$y_{14} = \frac{L^3}{EI} \left[-\frac{3}{250} Z + \frac{36}{1875} 0 - \frac{34}{1875} P - \frac{4}{375} T \right]$$

$$y_{15} = \frac{L^3}{EI} \left[-\frac{4}{375} Z + \frac{68}{3750} 0 - \frac{12}{625} P - \frac{3}{250} T \right]$$

$$y_{16} = \frac{L^3}{EI} \left[-\frac{23}{3750} Z + \frac{4}{375} 0 - \frac{9}{750} P - \frac{16}{1875} T \right]$$

$$\frac{68}{3750} F - \frac{68}{3750} G + \frac{24}{625} H + \frac{4}{375} J + \frac{3}{250} P + \frac{3}{250} S + \frac{4}{375} Y = 0$$

Nº 12 :

$$\frac{4}{375} Q + \frac{68}{3750} R + \frac{12}{625} S + \frac{3}{250} T = - \frac{23}{3750} Y + \frac{4}{375} G - \frac{9}{750} H - \frac{16}{1875} S$$

$$-\frac{4}{375} G + \frac{9}{750} H + \frac{4}{375} Q + \frac{68}{3750} R + \frac{52}{1875} S + \frac{3}{250} T + \frac{23}{3750} Y = 0$$

Nº 13 :

$$\frac{23}{3750} X + \frac{4}{375} E + \frac{9}{750} Y + \frac{16}{1875} Z = - \frac{16}{1875} Z + \frac{3}{250} O - \frac{4}{375} P - \frac{23}{3750} T$$

$$\frac{4}{375} E - \frac{3}{250} O + \frac{4}{375} P + \frac{23}{3750} T + \frac{23}{3750} X + \frac{9}{750} Y + \frac{32}{1875} Z = 0$$

Nº 14 :

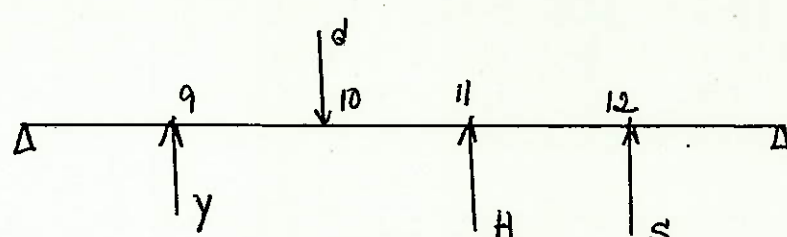
$$-\frac{23}{3750} I + \frac{4}{375} K - \frac{9}{750} G - \frac{16}{1875} O = - \frac{3}{250} Z + \frac{36}{1875} O - \frac{34P}{1875} - \frac{4}{375} T$$

$$-\frac{9}{750} G - \frac{23}{3750} I + \frac{4}{375} K - \frac{52}{1875} O + \frac{34}{1875} P + \frac{4}{375} T + \frac{3}{250} Z = 0$$

Nº 15 :

$$\frac{23}{3750} J + \frac{4}{375} F + \frac{9}{750} H + \frac{16}{1875} P = - \frac{4}{375} Z + \frac{68}{3750} O - \frac{12}{625} P - \frac{3}{250} T$$

VIGA II



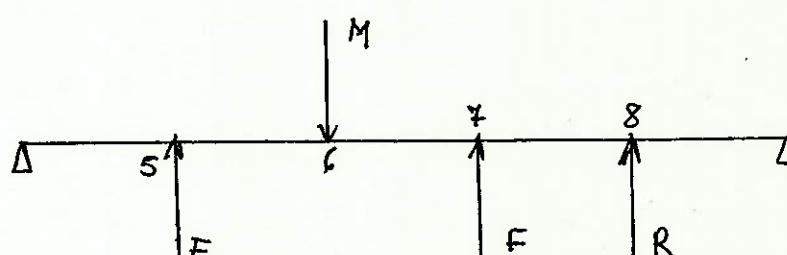
$$y_9 = \frac{L^3}{EI} \left[-\frac{16}{1875} Y + \frac{3}{250} G - \frac{4}{375} H - \frac{23}{3750} S \right]$$

$$y_{10} = \frac{L^3}{EI} \left[-\frac{3}{250} Y + \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S \right]$$

$$y_{11} = \frac{L^3}{EI} \left[-\frac{4}{375} Y + \frac{68}{3750} G - \frac{12}{625} H - \frac{3}{250} S \right]$$

$$y_{12} = \frac{L^3}{EI} \left[-\frac{23}{3750} Y + \frac{4}{375} G - \frac{9}{750} H - \frac{16}{1875} S \right]$$

VIGA III



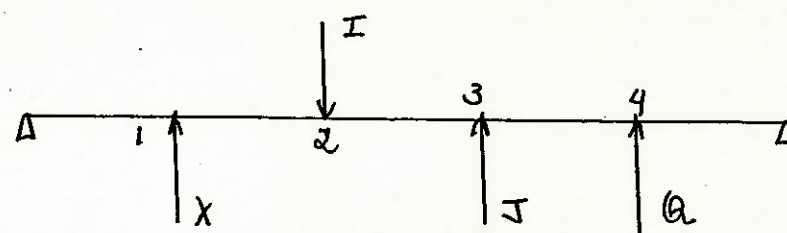
$$y_5 = \frac{L^3}{EI} \left[-\frac{16}{1875} E + \frac{3}{250} M - \frac{4}{375} F - \frac{23}{3750} R \right]$$

$$y_6 = \frac{L^3}{EI} \left[-\frac{3}{250} E + \frac{36}{1875} M - \frac{34}{1875} F - \frac{4}{375} R \right]$$

$$y_7 = \frac{L^3}{EI} \left[-\frac{4}{375} E + \frac{68}{3750} M - \frac{12}{625} F - \frac{3}{250} R \right]$$

$$y_8 = \frac{L^3}{EI} \left[-\frac{23}{3750} E + \frac{4}{375} M - \frac{9}{750} F - \frac{16}{1875} R \right]$$

VIGA IV



$$y_1 = \frac{L^3}{EI} \left[-\frac{16}{1875} X + \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q \right]$$

$$y_2 = \frac{L^3}{EI} \left[-\frac{3}{250} X + \frac{36}{1875} I - \frac{34}{1875} J - \frac{4}{375} Q \right]$$

$$y_3 = \frac{L^3}{EI} \left[-\frac{4}{375} X + \frac{68}{3750} I - \frac{12}{625} J - \frac{3}{250} Q \right]$$

$$y_4 = \frac{L^3}{EI} \left[-\frac{23}{3750} X + \frac{4}{375} I - \frac{9}{750} J - \frac{16}{1875} Q \right]$$

————— " ————— "

IGUALANDO - SE AS FLECHAS RESPECTIVAS ::

NÓ 1 :

$$\frac{16}{1875} X + \frac{3}{250} I + \frac{4}{375} J + \frac{23}{3750} Q = -\frac{16}{1875} X + \frac{3}{250} I - \frac{4}{375} J - \frac{23}{3750} Q$$

$$\frac{3}{250} I - \frac{3}{250} I + \frac{4}{375} J + \frac{23}{3750} Q + \frac{32}{1875} X + \frac{4}{375} J + \frac{23}{3750} Q = 0$$

NÓ 2 :

$$-\frac{16}{1875} I + \frac{3}{250} K - \frac{4}{375} G - \frac{23}{3750} O = -\frac{3}{250} X + \frac{36}{1875} I - \frac{34J}{1875} - \frac{4Q}{375}$$

$$-\frac{52}{1875}I + \frac{34}{1875}J + \frac{3}{250}K + \frac{4}{375}Q - \frac{4}{375}G - \frac{23}{3750}O + \frac{3}{250}X = 0$$

$$-\frac{4}{375}G - \frac{52}{1875}I + \frac{34}{1875}J + \frac{3}{250}K - \frac{23}{3750}O + \frac{4}{375}Q + \frac{3}{250}X = 0$$

Nº 3 :

$$\frac{16}{1875}J + \frac{3}{250}F + \frac{4}{375}H + \frac{23}{3750}P = -\frac{4}{375}X + \frac{68}{3750}I - \frac{12}{625}J - \frac{3}{250}Q$$

$$\frac{3}{250}F + \frac{4}{375}H - \frac{68}{3750}I + \frac{52}{1875}J + \frac{23}{3750}P + \frac{3}{250}Q + \frac{4}{375}X = 0$$

Nº 4 :

$$\frac{16}{1875}Q + \frac{3}{250}R + \frac{4}{375}S + \frac{23}{3750}T = -\frac{23}{3750}X + \frac{4}{375}I - \frac{9}{750}J - \frac{16}{1875}Q$$

$$-\frac{4}{375}I + \frac{9}{750}J + \frac{32}{1875}Q + \frac{3}{250}R + \frac{4}{375}S + \frac{23}{3750}T + \frac{23}{3750}X = 0$$

Nº 5 :

$$\frac{3}{250}X + \frac{36}{1875}E + \frac{34}{1875}Y + \frac{4}{375}Z = -\frac{16}{1875}E + \frac{3}{250}M - \frac{4}{375}F - \frac{23}{3750}R$$

$$\frac{52}{1875}E + \frac{4}{375}F - \frac{3}{250}M + \frac{23}{3750}R + \frac{3}{250}X + \frac{34}{1875}Y + \frac{4}{375}Z = 0$$

$$\frac{52}{1875} E + \frac{4}{375} F + \frac{3}{250} K + \frac{23}{3750} R + \frac{3}{250} X + \frac{34}{1875} Y + \frac{4}{375} Z = \frac{3}{250}$$

Nº 6 :

$$-\frac{3}{250} I + \frac{36}{1875} K - \frac{34}{1875} G - \frac{4}{375} O = -\frac{3}{250} E + \frac{36}{1875} M - \frac{34}{1875} F - \frac{4}{375} R$$

$$\frac{3}{250} E + \frac{34}{1875} F - \frac{34}{1875} G - \frac{3}{250} I + \frac{36}{1875} K - \frac{36}{1875} M - \frac{4}{375} O + \frac{4}{375} R = 0$$

$$M = 1 - K \implies -\frac{36}{1875} (1 - K) = -\frac{36}{1875} + \frac{36}{1875} K$$

$$\frac{3}{250} E + \frac{34}{1875} F - \frac{34}{1875} G - \frac{3}{250} I + \frac{72}{1875} K - \frac{4}{375} O + \frac{4}{375} R = \frac{36}{1875}$$

Nº 7 :

$$\frac{3}{250} J + \frac{36}{1875} F + \frac{34}{1875} H + \frac{4}{375} P = -\frac{4}{375} E + \frac{68}{3750} M - \frac{12}{625} F - \frac{3}{250} R$$

$$\frac{4}{375} E + \frac{72}{1875} F + \frac{34}{1875} H + \frac{3}{250} J - \frac{68}{3750} M + \frac{4}{375} P + \frac{3}{250} R = 0$$

$$-\frac{68}{3750} M = -\frac{68}{3750} (1 - K) = -\frac{68}{3750} + \frac{68}{3750} K$$

$$\frac{4}{375} E + \frac{72}{1875} F + \frac{34}{1875} H + \frac{3}{250} J + \frac{68}{3750} K + \frac{4}{375} P + \frac{3}{250} R = \frac{68}{3750}$$

N^o 8 :

$$\frac{3}{250} Q + \frac{36}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = -\frac{23}{3750} E + \frac{4}{375} M - \frac{9}{750} F - \frac{16}{1875} R$$

$$\frac{23}{3750} E + \frac{9}{750} F - \frac{4}{375} M + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = 0$$

$$-\frac{4}{375} M = -\frac{4}{375} (1 - K) = -\frac{4}{375} + \frac{4}{375} K$$

$$\frac{23}{3750} E + \frac{9}{750} F + \frac{4}{375} K + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = \frac{4}{375}$$

N^o 9 :

$$\frac{4}{375} X + \frac{68}{3750} E + \frac{12}{625} Y + \frac{3}{250} Z = -\frac{16}{1875} Y + \frac{3}{250} G - \frac{4}{375} H - \frac{23}{3750} S$$

$$\frac{68}{3750} E - \frac{3}{250} G + \frac{4}{375} H + \frac{23}{3750} S + \frac{4}{375} X + \frac{52}{1875} Y + \frac{3}{250} Z = 0$$

N^o 10 :

$$-\frac{4}{375} I + \frac{68}{3750} K - \frac{12}{625} G - \frac{3}{250} O = -\frac{3}{250} Y + \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S$$

$$-\frac{72}{1875} G + \frac{34}{1875} H - \frac{4}{375} I + \frac{68}{3750} K - \frac{3}{250} O + \frac{4}{375} S + \frac{3}{250} Y = 0$$

N^o 11:

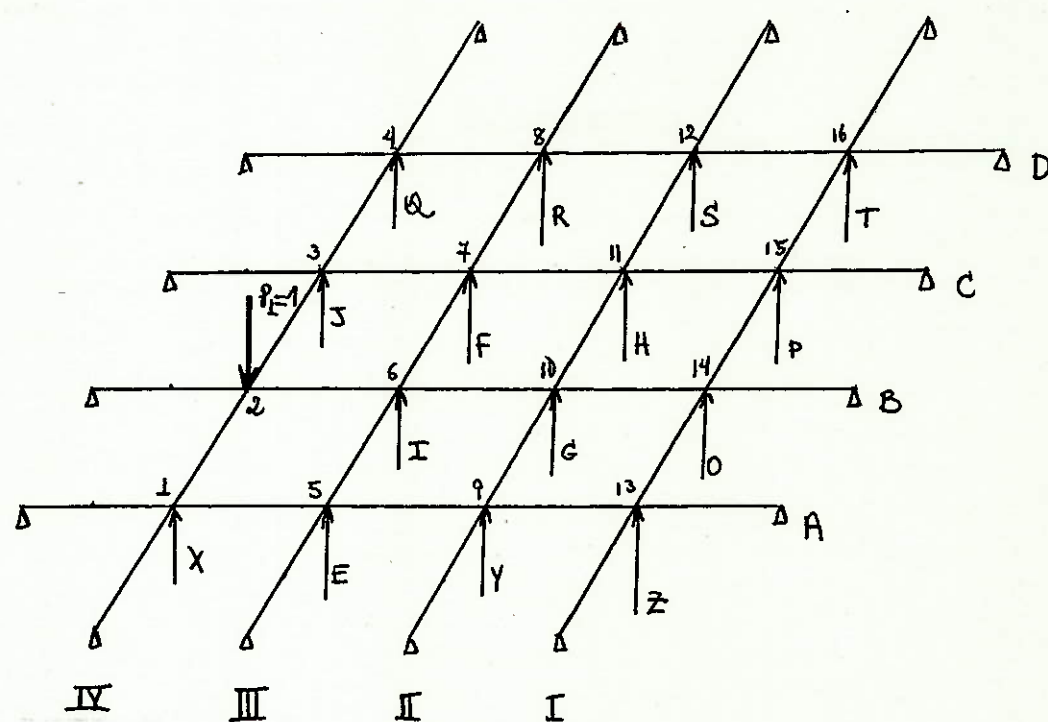
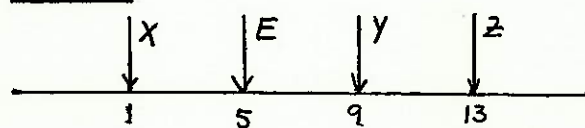
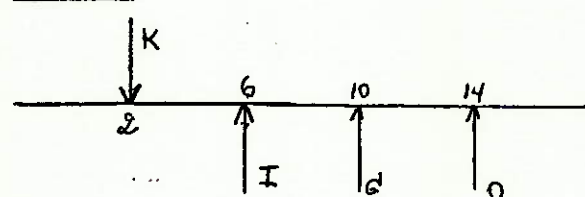
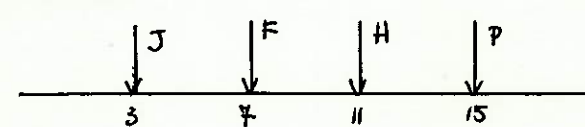
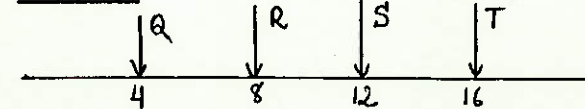
$$\frac{4}{375} J + \frac{68}{3750} F + \frac{12}{625} H + \frac{3}{250} P = -\frac{4}{375} Y + \frac{68}{3750} G - \frac{12}{625} H - \frac{3}{250} S$$

$$\frac{4}{375} F + \frac{9}{750} H + \frac{23}{3750} J - \frac{68}{3750} O + \frac{52}{1875} P + \frac{3}{250} T + \frac{4}{375} Z = 0$$

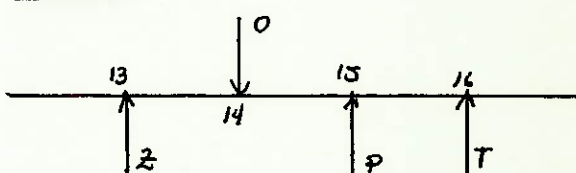
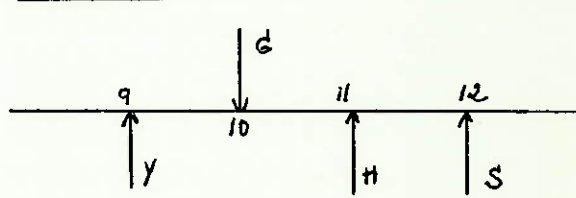
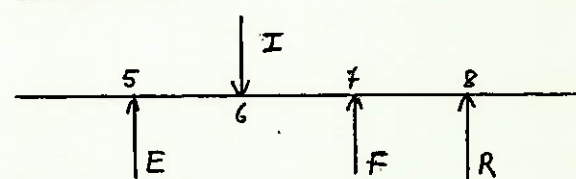
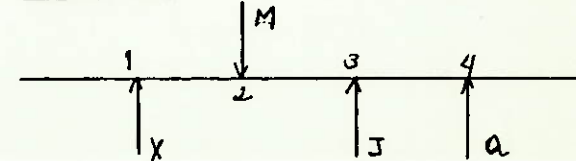
Nº 16 ::

$$\frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{3750} S + \frac{16}{1875} T = - \frac{23}{3750} Z + \frac{4}{375} O - \frac{9}{750} P - \frac{16}{1875} T$$

$$-\frac{4}{375} O + \frac{9}{750} P + \frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{32}{1875} T + \frac{23}{3750} Z = 0$$

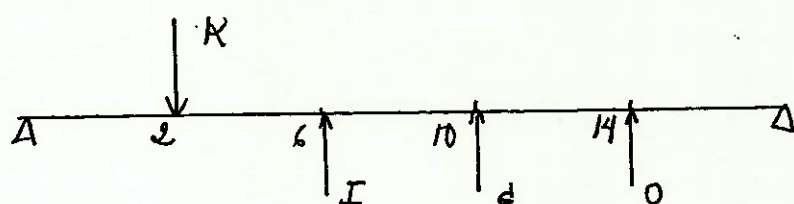
CASO DVIGA AVIGA BVIGA CVIGA D

$$\underline{K + M = 1}$$

VIGA IVIGA IIVIGA IIIVIGA IV

VIGA A:

igual ao caso C

VIGA B:

$$y_2 = \frac{L^3}{EI} \left[\frac{16}{1875} K - \frac{3}{250} I - \frac{4}{375} G - \frac{23}{3750} O \right]$$

$$y_6 = \frac{L^3}{EI} \left[\frac{3}{250} K - \frac{36}{1875} I - \frac{34}{1875} G - \frac{4}{375} O \right]$$

$$y_{10} = \frac{L^3}{EI} \left[\frac{4}{375} K - \frac{68}{3750} I - \frac{12}{625} G - \frac{3}{250} O \right]$$

$$y_{14} = \frac{L^3}{EI} \left[\frac{23}{3750} K - \frac{4}{375} I - \frac{9}{750} G - \frac{16}{1875} O \right]$$

VIGA C:

igual ao caso C

VIGA D:

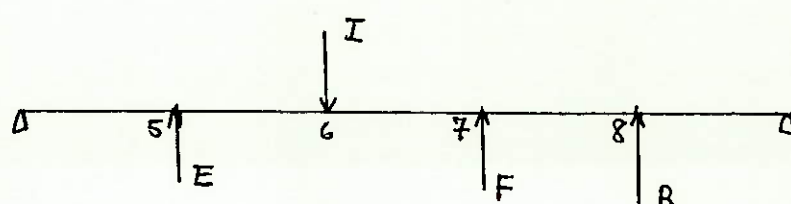
igual ao caso C

VIGA I:

igual ao caso C

VIGA II:

igual ao caso C

VIGA III:

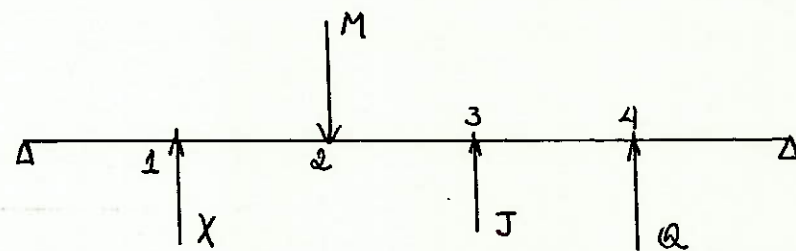
$$y_5 = \frac{L^3}{EI} \left[-\frac{16}{1875} E + \frac{3}{250} I - \frac{4}{375} F - \frac{23}{3750} R \right]$$

$$y_6 = \frac{L^3}{EI} \left[-\frac{3}{250} E + \frac{36}{1875} I - \frac{34}{1875} F - \frac{4}{375} R \right]$$

$$y_7 = \frac{L^3}{EI} \left[-\frac{4}{375} E + \frac{68}{3750} I - \frac{12}{625} F - \frac{3}{250} R \right]$$

$$y_8 = \frac{L^3}{EI} \left[-\frac{23}{3750} E + \frac{4}{375} I - \frac{9}{750} F - \frac{16}{1875} R \right]$$

VIGA LV :



$$y_1 = \frac{L^3}{EI} \left[-\frac{16}{1875} X + \frac{3}{250} M - \frac{4}{375} J - \frac{23}{3750} Q \right]$$

$$y_2 = \frac{L^3}{EI} \left[-\frac{3}{250} X + \frac{36}{1875} M - \frac{34}{1875} J - \frac{4}{375} Q \right]$$

$$y_3 = \frac{L^3}{EI} \left[-\frac{4}{375} X + \frac{68}{3750} M - \frac{12}{625} J - \frac{3}{250} Q \right]$$

$$y_4 = \frac{L^3}{EI} \left[-\frac{23}{3750} X + \frac{4}{375} M - \frac{9}{750} J - \frac{16}{1875} Q \right]$$

_____ " _____ "

IGUALANDO - SE AS FLECHAS RESPECTIVAS TEMOS :

NÓ 1 :

$$\frac{16}{1875} X + \frac{3}{250} E + \frac{4}{375} Y + \frac{23}{3750} Z = -\frac{16}{1875} X + \frac{3}{250} M - \frac{4}{375} J - \frac{23}{3750} Q$$

$$\frac{3}{250} E + \frac{4}{375} J - \frac{3}{250} M + \frac{23}{3750} Q + \frac{32}{1875} X + \frac{4}{375} Y + \frac{23}{3750} Z = 0$$

$$M = 1 - K$$

$$\frac{3}{250} E + \frac{4}{375} J + \frac{3}{250} K + \frac{23}{3750} Q + \frac{32}{1875} X + \frac{4}{375} Y + \frac{23}{3750} Z = \frac{3}{250}$$

Nº 2 :

$$\frac{16}{1875} K - \frac{3}{250} I - \frac{4}{375} G - \frac{23}{3750} O = -\frac{3}{250} X + \frac{36}{1875} M - \frac{34}{1875} J - \frac{4}{375} Q$$

$$-\frac{4}{375} G - \frac{3}{250} I + \frac{34}{1875} J + \frac{16}{1875} K - \frac{36}{1875} M - \frac{23}{3750} O + \frac{4}{375} Q + \frac{3}{250} X = 0$$

$$-\frac{4}{375} G - \frac{3}{250} I + \frac{34}{1875} J + \frac{52}{1875} K - \frac{23}{3750} O + \frac{4}{375} Q + \frac{3}{250} X = \frac{36}{1875}$$

Nº 3 :

$$\frac{16}{1875} J + \frac{3}{250} F + \frac{4}{375} H + \frac{23}{3750} P = -\frac{4}{375} X + \frac{68}{3750} M - \frac{12}{625} J - \frac{3}{250} Q$$

$$\frac{3}{250} F + \frac{4}{375} H + \frac{52}{1875} J - \frac{68}{3750} M + \frac{23}{3750} P + \frac{3}{250} Q + \frac{4}{375} X = 0$$

$$\frac{3}{250} F + \frac{4}{375} H + \frac{52}{1875} J + \frac{68}{3750} K + \frac{23}{3750} P + \frac{3}{250} Q + \frac{4}{375} X = \frac{68}{3750}$$

Nº 4 :

$$\frac{16}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T = -\frac{23}{3750} X + \frac{4}{375} M - \frac{9}{750} J - \frac{16}{1875} Q$$

$$\frac{9}{750} J - \frac{4}{375} M + \frac{32}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T + \frac{23}{3750} X = 0$$

$$\frac{9}{750} J + \frac{4}{375} K + \frac{32}{1875} Q + \frac{3}{250} R + \frac{4}{375} S + \frac{23}{3750} T + \frac{23}{3750} X = \frac{4}{375}$$

Nº 5 :

$$\frac{3}{250} X + \frac{36}{1875} E + \frac{34}{1875} Y + \frac{4}{375} Z = \frac{-16}{1875} E + \frac{3}{250} I - \frac{4}{375} F - \frac{23}{3750} R$$

$$\frac{52}{1875} E + \frac{4}{375} F - \frac{3}{250} I + \frac{23}{3750} R + \frac{3}{250} X + \frac{34}{1875} Y + \frac{4}{375} Z = 0$$

Nº 6 :

$$\frac{3}{250} K - \frac{36}{1875} I - \frac{34}{1875} G - \frac{4}{375} O = -\frac{3}{250} E + \frac{36}{1875} I - \frac{34}{1875} F - \frac{4}{375} R$$

$$\frac{3}{250} E + \frac{34}{1875} F - \frac{34}{1875} G - \frac{72}{1875} I + \frac{3}{250} K - \frac{4}{375} O + \frac{4}{375} R = 0$$

Nº 7 :

$$\frac{3}{250} J + \frac{36}{1875} F + \frac{34}{1875} H + \frac{4}{375} P = -\frac{4}{375} E + \frac{68}{3750} I - \frac{12}{625} F - \frac{3R}{250}$$

$$\frac{4}{375} E + \frac{72}{1875} F + \frac{34}{1875} H - \frac{68}{3750} I + \frac{3}{250} J + \frac{4}{375} P + \frac{3}{250} R = 0$$

N^o 8 :

$$\frac{3}{250} Q + \frac{36}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = - \frac{23}{3750} E + \frac{4}{375} I - \frac{9}{750} F - \frac{16}{1875} R$$

$$\frac{23}{3750} E + \frac{9}{750} F - \frac{4}{375} I + \frac{3}{250} Q + \frac{52}{1875} R + \frac{34}{1875} S + \frac{4}{375} T = 0$$

N^o 9 :

$$\frac{68}{3750} E - \frac{3}{250} G + \frac{4}{375} H + \frac{23}{3750} S + \frac{4}{375} X + \frac{52}{1875} Y + \frac{3}{250} Z = 0$$

N^o 10:

$$- \frac{3}{250} Y + \frac{36}{1875} G - \frac{34}{1875} H - \frac{4}{375} S = \frac{4}{375} K - \frac{68}{3750} I - \frac{12G}{625} - \frac{30}{250}$$

$$\frac{72}{1875} G - \frac{34}{1875} H + \frac{68}{3750} I - \frac{4}{375} K + \frac{3}{250} O - \frac{4}{375} S - \frac{3}{250} Y = 0$$

N^o 11 :

$$\frac{68}{3750} F - \frac{68}{3750} G + \frac{24}{625} H + \frac{4}{375} J + \frac{3}{250} P + \frac{3}{250} S + \frac{4}{375} Y = 0$$

N^o 12 :

$$- \frac{4}{375} F + \frac{9}{750} H + \frac{4}{375} Q + \frac{68}{3750} R + \frac{52}{1875} S + \frac{3}{250} T + \frac{23}{3750} Y = 0$$

Nº 13 :

$$\frac{4}{375} E - \frac{3}{250} O + \frac{4}{375} P + \frac{23}{3750} T + \frac{23}{3750} X + \frac{9}{750} Y + \frac{32}{1875} Z = 0$$

Nº 14 :

$$-\frac{3}{250} Z + \frac{36}{1875} O - \frac{34}{1875} P - \frac{4}{375} T = \frac{23}{3750} K - \frac{4}{375} I - \frac{9}{750} G - \frac{16}{1875} O$$

$$\frac{9}{750} G + \frac{4}{375} I - \frac{23}{3750} K + \frac{52}{1875} O - \frac{34}{1875} P - \frac{4}{375} T - \frac{3}{250} Z = 0$$

Nº 15 :

$$\frac{4}{375} F + \frac{9}{750} H + \frac{23}{3750} J - \frac{68}{3750} O + \frac{52}{1875} P + \frac{3}{250} T + \frac{4}{375} Z = 0$$

Nº 16 :

$$-\frac{4}{375} O + \frac{9}{750} P + \frac{23}{3750} Q + \frac{4}{375} R + \frac{9}{750} S + \frac{32}{1875} T + \frac{23}{3750} Z = 0$$

V) SOLUÇÕES DOS SISTEMAS

A resolução dos sistemas dos casos A , B , C , D foi executada pela subrotina " LEQTIF " , da Biblioteca / do Centro de Computação Eletrônica da USP , na qual se entra com as matrizes na forma $A.X=B$, onde :

A = matriz dos coeficientes das incógnitas(16x16)

B = matriz das constantes (16x1)

X = matriz das incógnitas

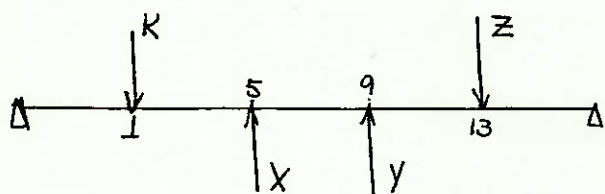
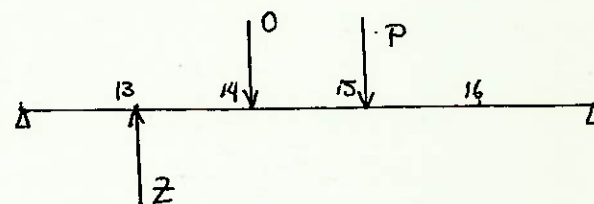
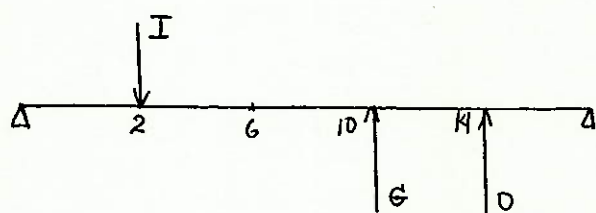
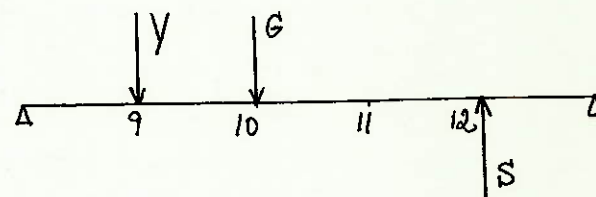
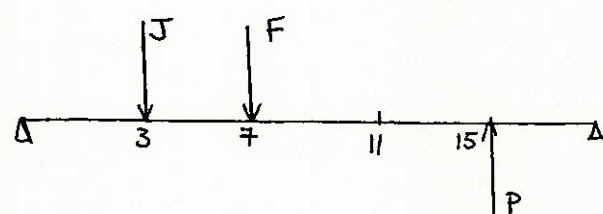
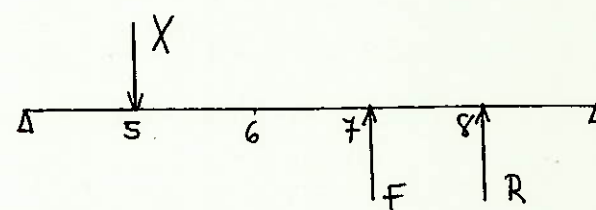
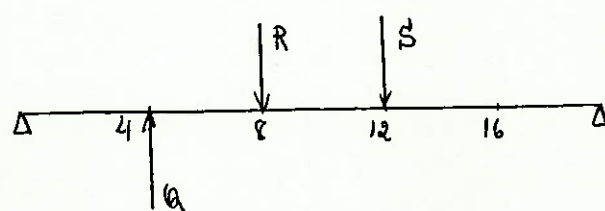
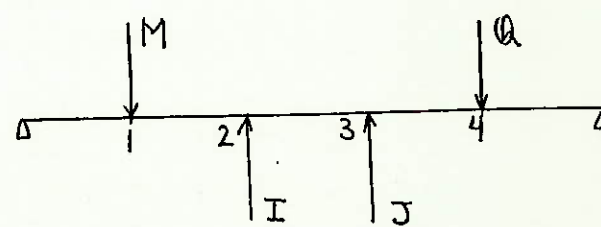
Essa subrotina resolve o sistema de equações lineares pelo método de eliminação de Gauss , com pivotamento parcial .

Para maiores detalhes , consultar o anexo e as / referências bibliográficas .

CASO A

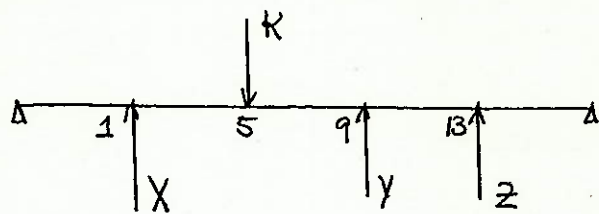
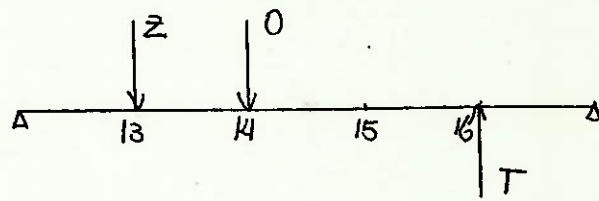
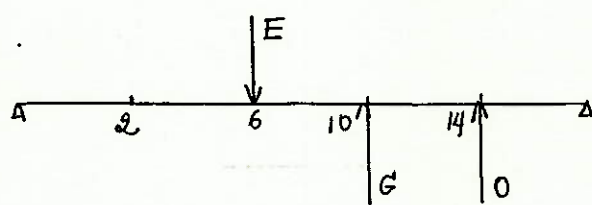
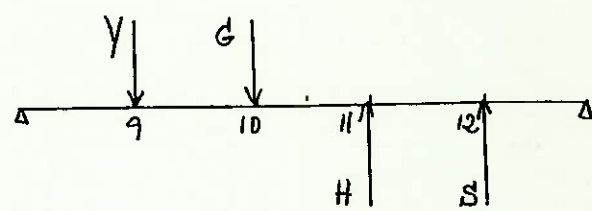
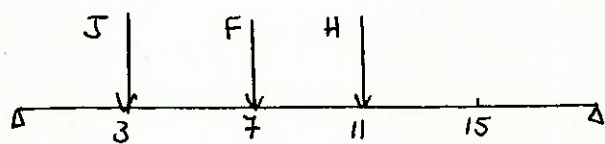
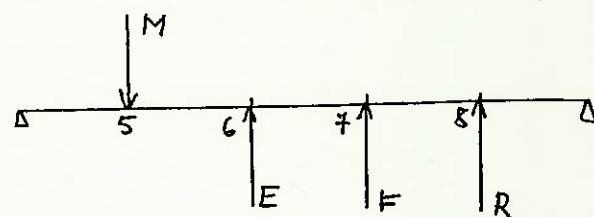
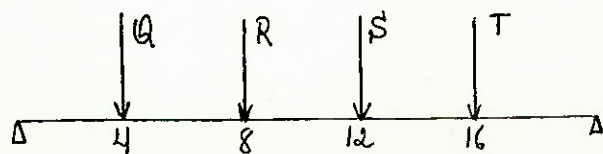
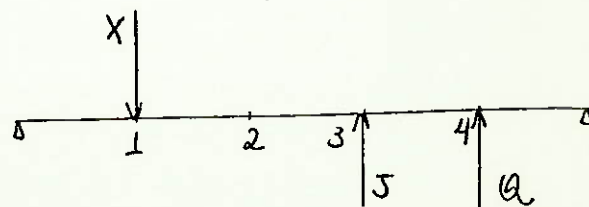
E = 0,0000000
F = 0,0506755
G = 0,0506755
H = 0,0000000
I = 0,2202332
J = 0,0299199
K = 0,5000000
O = -0,0171034
P = -0,0151012
Q = -0,0004598
R = 0,0171034
S = 0,0151012
T = 0,0000000
X = 0,2202332
Y = 0,0299199
Z = -0,0004598

$$K + M = 1 \quad . \cdot . \quad M = 0,5000000$$

VIGA AVIGA IVIGA BVIGA IIVIGA CVIGA IIIVIGA DVIGA IV

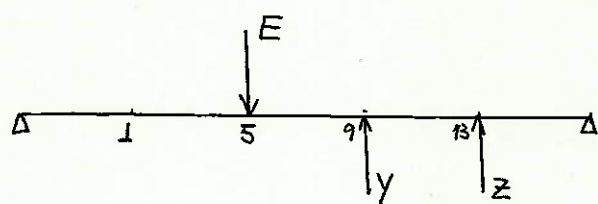
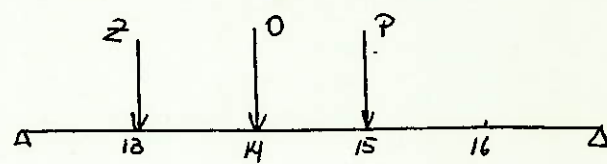
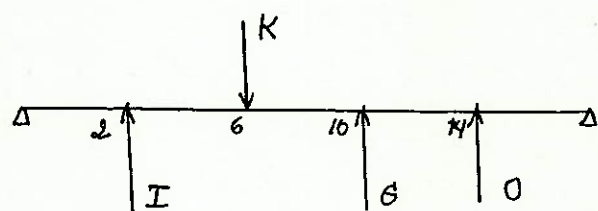
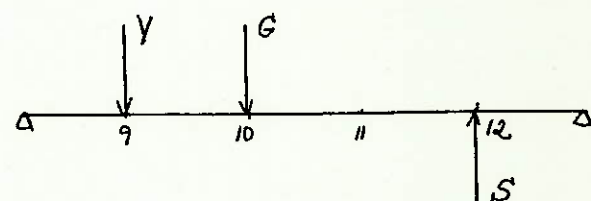
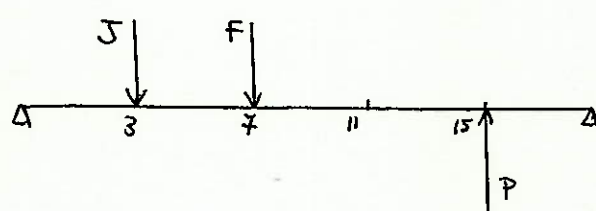
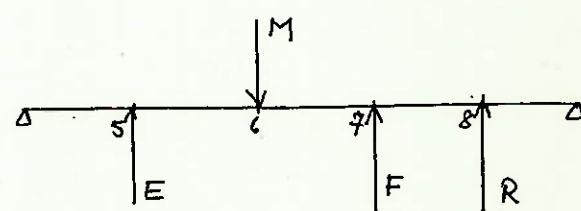
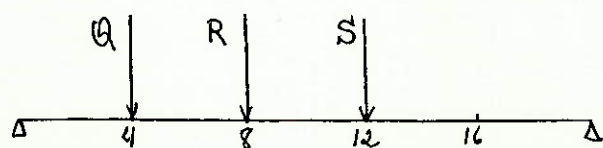
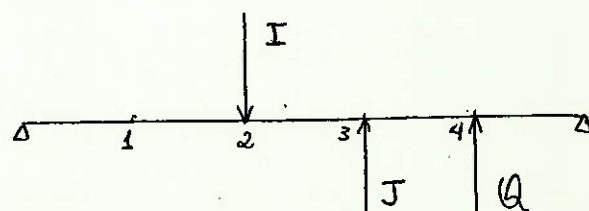
CASO B

E	=	0,1695595
F	=	0,0299219
G	=	-0,0171054
H	=	0,0355725
I	=	0,0000000
J	=	0,0506755
K	=	0,4700724
O	=	-0,0506755
P	=	0,0000000
Q	=	0,0171034
R	=	0,0146416
S	=	0,0171034
T	=	0,0151012
X	=	0,2202332
Y	=	0,2197668
Z	=	0,0299199
M	=	0,5299276

VIGA AVIGA IVIGA BVIGA IIVIGA CVIGA IIIVIGA DVIGA IV

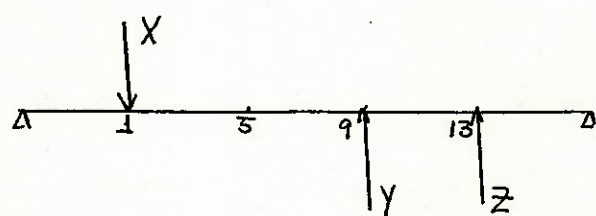
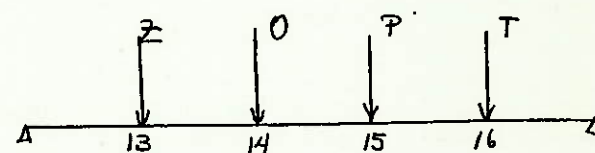
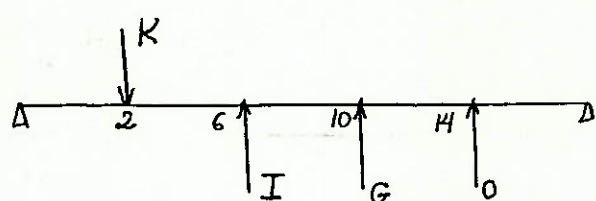
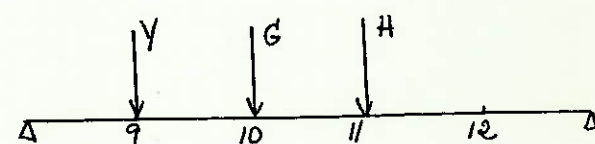
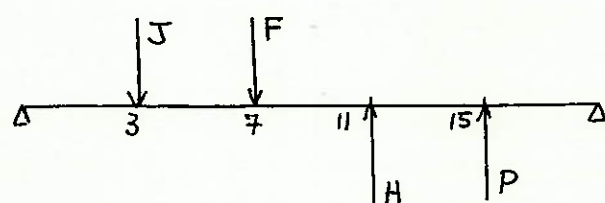
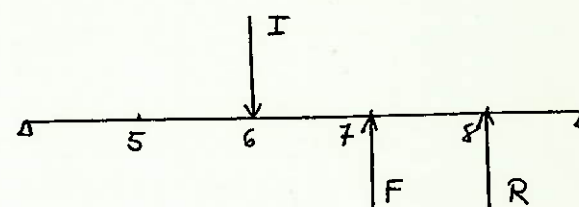
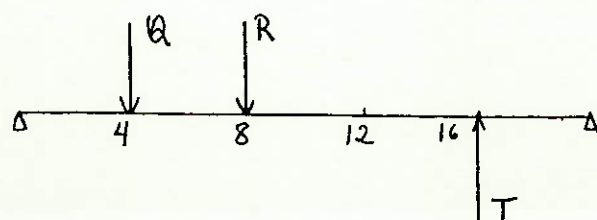
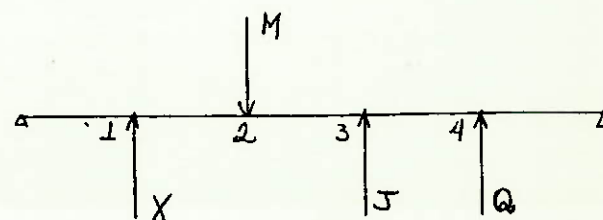
CASO C

E	=	0,1695595
F	=	0,1841963
G	=	0,1841963
H	=	0,0000000
I	=	0,1695595
J	=	0,0171054
K	=	0,5000000
O	=	0,0299219
P	=	0,0355725
Q	=	0,0506755
R	=	0,0299219
S	=	0,0355725
T	=	0,0000000
X	=	0,0000000
Y	=	-0,0171054
Z	=	-0,0506755
M	=	0,5000000

VIGA AVIGA IVIGA BVIGA IIVIGA CVIGA IIIVIGA DVIGA IV

CASO D

E = 0,0000000
F = 0,0171054
G = 0,0299219
H = -0,0355725
I = 0,1695595
J = 0,2197668
K = 0,4700724
O = 0,0146416
P = -0,0171034
Q = 0,0299199
R = 0,0506755
S = 0,0000000
T = -0,0151012
X = 0,2202332
Y = -0,0506755
Z~~z~~ = -0,0171034
M = 0,5299276

VIGA AVIGA IVIGA BVIGA IIVIGA CVIGA IIIVIGA DVIGA IV

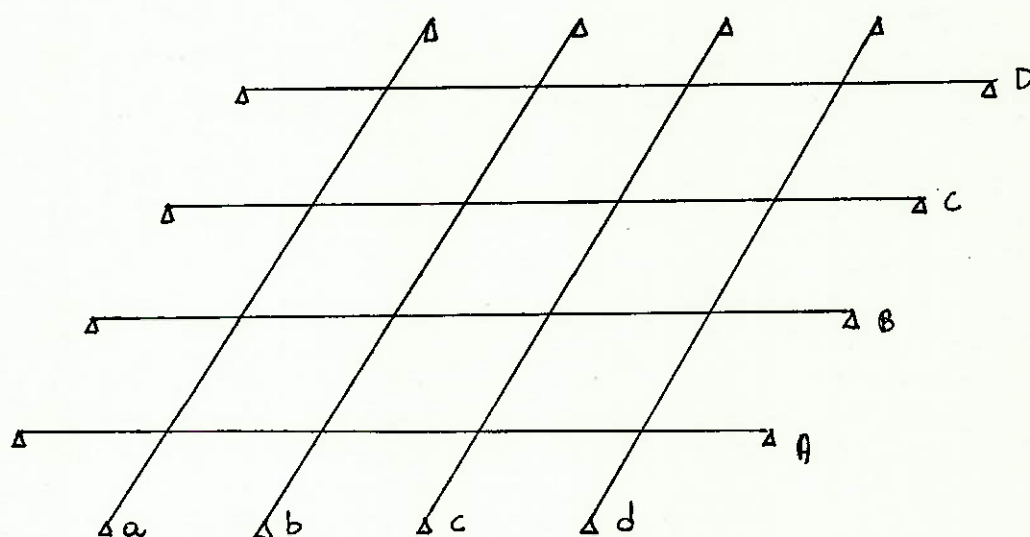
VI- CONCLUSÕES

Das soluções apresentadas no capítulo V, podem ser confeccionadas tabelas práticas para uso rápido, sem entrar no mérito da teoria, tabelas essas de Reações, Forças Cortantes e Momentos.

Para a confecção dessas tabelas, passaremos a uma mudança de nomenclatura usada até agora, para uma facilidade maior no uso e manuseio das mesmas.

A nomenclatura anterior foi usada tendo em vista compatibilizar os quatro casos com o programa do computador.

Todavia agora, de posse dos resultados, passaremos a adotar a seguinte nomenclatura:



Os nós serão designados pelas intersecções das letras, isto é, R_{aA} será a reação no nó aA, e assim sucessivamente.

Teremos também:

Caso I- esforço aplicado no nó aA

Caso II- esforço aplicado no nó bA

Caso III- esforço aplicado no nó bB

Caso IV - esforço aplicado no nó aB

Podemos construir a seguinte tabela para o Caso I :

Reações ($\times 10^3$)	Caso I - Esforço + P ($\downarrow$) aplicado em "aA"							
	Vigas (multiplicar as reações por P)							
	A	B	C	D	a	b	c	d
R_{aA}	+ 500,000				+ 500,000			
R_{aB}		+ 220,2332			- 220,2332			
R_{aC}			+ 29,9199		- 29,9199			
R_{aD}				- 0,4598	+ 0,4598			
R_{bA}	- 220,2332					+ 220,2332		
R_{bB}		0,0000				0,0000		
R_{bC}			+ 50,6755			- 50,6755		
R_{bD}				+ 17,1034		- 17,1034		
R_{cA}	- 29,9199						+ 29,9199	
R_{cB}		- 50,6755					+ 50,6755	
R_{cC}			0,0000				0,0000	
R_{cD}				+ 15,1012			- 15,1012	
R_{dA}	+ 0,4598							- 0,4598
R_{dB}		- 17,1034						+ 17,1034
R_{dC}			- 15,1012					+ 15,1012
R_{dD}				0,0000				0,0000

Exemplo: queremos saber as reações na viga C, quando se aplica a carga $P=2\text{tf}$ no nó aA.

solução: da tabela temos para a viga C:

$$R_{aC} = 29,919 \cdot 10^{-3}$$

$$R_{bC} = 50,6755 \cdot 10^{-3}$$

$$R_{cC} = 0,0000000$$

$$R_{dC} = -15,1012 \cdot 10^{-3}$$

e, multiplicando as reações por 2 tf teremos as reações procuradas:

$$R'_{aC} = 5,9838 \cdot 10^{-2} \text{ tf}$$

$$R'_{bC} = 1,01351 \cdot 10^{-1} \text{ tf}$$

$$R'_{cC} = 0,00000000 \text{ tf}$$

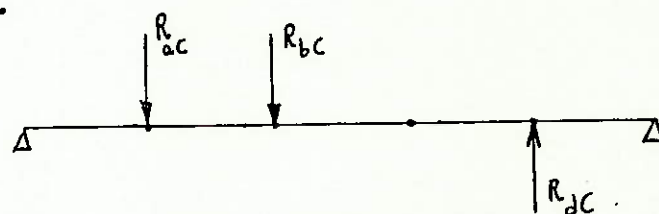
$$R'_{dC} = -3,02024 \cdot 10^{-2} \text{ tf}$$

De maneira análoga, podemos construir as tabelas de reações para os casos II, III e IV.

Notar que os casos II e IV são simétricos.

As tabelas de Forças Cortantes e Momentos Fletores podem ser construídas fazendo-se um estudo das vigas no tocante a esses parâmetros.

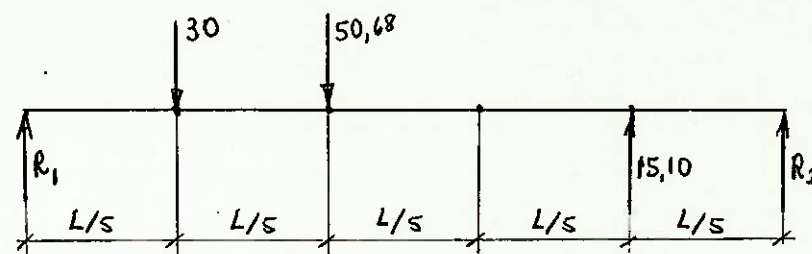
Suponhamos ainda o caso I, e consideremos a viga C para exemplificar.



A menos de 10^{-3} temos: $R_{aC} = 30,00$

$$R_{bC} = 50,69$$

$$R_{dC} = 15,10$$



Daí $R_2 = 14,19$

$R_1 = 51,39$

Diagrama de Forças Cortantes

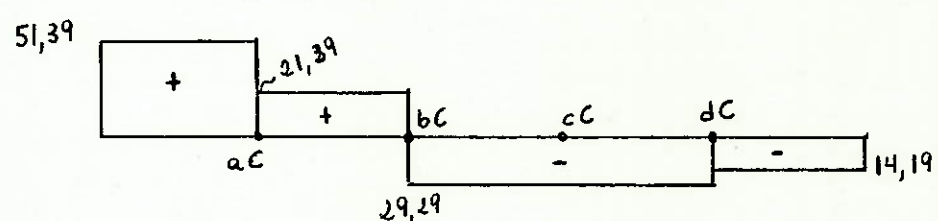
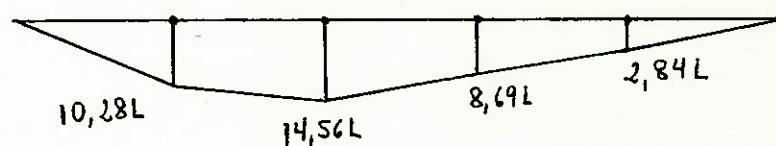


Diagrama de Momentos Fletores



Nota-se que a linha de força cortante é constante entre nós consecutivos.

Nota-se também que a linha de Momentos é linear entre nós consecutivos, podendo-se portanto interpolar linearmente entre os nós.

Dessa forma, poderíamos construir tabelas de Momentos -

nos nós para todas as vigas, assim como tabelas de Forças Cortantes para todos os trechos de todas as vigas, para os casos I, II, III e IV.

Essas tabelas seriam semelhantes ao modelo anterior, substituindo a coluna de reações por Momentos e Forças Cortantes.

Com relação à Força Cortante, a tabela teria que abranger o trecho entre dois nós consecutivos, já que entre os nós a força cortante é constante.

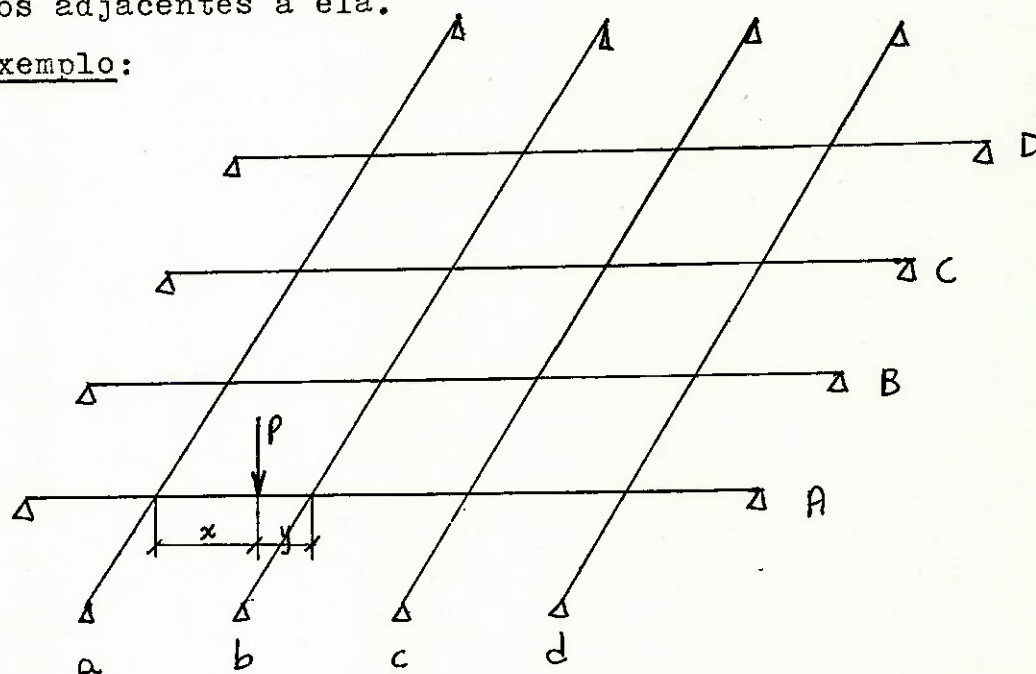
Outro fato é que, quando temos, por exemplo, cargas aplicadas nos nós aA e bA, podemos fazer a superposição das tabelas do Caso I e Caso II.

E assim com outros casos semelhantes.

Assim, teremos o resultado procurado.

Também quando se tem uma força aplicada entre nós, sobre uma barra, podemos decompô-la e dividir seu efeito entre os 2 nós adjacentes a ela.

Exemplo:



$$P_{bA} = \frac{Px}{L/5}$$

$$P_{aA} = \frac{Py}{L/5}$$

Assim, comporíamos os Casos I e II e solucionaríamos o problema.

VII- Referências Bibliográficas

- 1- Resistência dos Materiais - S. Timoshenko
- 2- Curso de Fortran IV - Setzer, Simon, Kowaltowski
- 3- Introdução ao Cálculo Numérico- Ivan de Queiroz Barros
- 4- Computer Solution of Linear Algebraic Systems-
George and Moler Forsythe.

\$

ANEXOS

IMSL ROUTINE NAME - LEQTLF

PURPOSE - LINEAR EQUATION SOLUTION - FULL STORAGE
MODE - SPACE ECONOMIZER SOLUTION.

USAGE - CALL LEQTLF (A,M,N,IA,B,IDGT,WKAREA,IER)

ARGUMENTS

A	- INPUT MATRIX OF DIMENSION N BY N CONTAINING THE COEFFICIENT MATRIX OF THE EQUATION $AX = B$. ON OUTPUT, A IS REPLACED BY THE LU DECOMPOSITION OF A ROWWISE PERMUTATION OF A.
M	- NUMBER OF RIGHT-HAND SIDES. (INPUT)
N	- ORDER OF A AND NUMBER OF ROWS IN B. (INPUT)
IA	- ROW DIMENSION OF A AND B EXACTLY AS SPECIFIED IN THE DIMENSION STATEMENT OF THE CALLING PROGRAM. (INPUT)
B	- INPUT MATRIX OF DIMENSION N BY M CONTAINING RIGHT-HAND SIDES OF THE EQUATION $AX = B$. ON OUTPUT, THE N BY M SOLUTION X REPLACES B.
IDGT	- INPUT OPTION. IF IDGT IS GREATER THAN 0, THE ELEMENTS OF A AND B ARE ASSUMED TO BE CORRECT TO IDGT DECIMAL DIGITS AND THE ROUTINE PERFORMS AN ACCURACY TEST. IF IDGT EQUALS ZERO, THE ACCURACY TEST IS BYPASSED.
WKAREA	- WORK AREA OF DIMENSION GREATER THAN OR EQUAL TO N.
IER	- ERROR PARAMETER. (OUTPUT) TERMINAL ERROR IER = 129 INDICATES THAT MATRIX A IS ALGORITHMICALLY SINGULAR. (SEE THE CHAPTER L PRELUDE). WARNING ERROR IER = 34 INDICATES THAT THE ACCURACY TEST FAILED. THE COMPUTED SOLUTION MAY BE IN ERROR BY MORE THAN CAN BE ACCOUNTED FOR BY THE UNCERTAINTY OF THE DATA. THIS WARNING CAN BE PRODUCED ONLY IF IDGT IS GREATER THAN 0 ON INPUT. (SEE CHAPTER L PRELUDE FOR FURTHER DISCUSSION).

PRECISION/HARDWARE - SINGLE AND DOUBLE/H32
- SINGLE/H36,H48,H60

REQD. IMSL ROUTINES - LUDATF,LUELMF,UERTST,UGETIO

NOTATION - INFORMATION ON SPECIAL NOTATION AND
CONVENTIONS IS AVAILABLE IN THE MANUAL
INTRODUCTION OR THROUGH IMSL ROUTINE UHELP

Algorithm

LEQTLF solves the set of linear equations $AX=B$ for X , given the N by N matrix A (in full storage mode). The N by M solution X overwrites B . The chief advantage of this program is that it uses less space than LEQT2F.

This routine performs Gaussian elimination (Crout algorithm) with equilibration and partial pivoting.

See reference: Forsythe, George and Moler, Cleve B., Computer Solution of Linear Algebraic Systems, Englewood Cliffs, N. J., Prentice-Hall, Inc., 1967, Chapter 9.

Note: This routine uses a Crout algorithm for decomposition (as opposed to the elimination routine in the above reference).

Accuracy

If IDGT is greater than zero, elements of A are assumed to be correct to IDGT decimal digits. The solution X will be the exact solution - without any roundoff error - to a matrix $\bar{A}$ which agrees with A in the first IDGT decimal digits. If such a solution is not obtainable in the accuracy with which we are working, a warning is given (IER=34). If IDGT equals zero, this accuracy test is bypassed.

Example

This example solves the set of linear equations $AX=B$ for X where A is a 3 by 3 matrix and B is a 3 by 4 matrix. Since IDGT $\neq$ 0, an accuracy test is performed.

Input:

```
REAL A(4,4),B(4,4),WKAREA(16)
INTEGER M,N,IA,IDGT,IER
```

```
N      =    3
M      =    4
IA     =    4
IDGT   =    3
```

```
A      =  [ 33.000   16.0   72.0   x ]
           [ -24.000  -10.0  -57.0   x ]
           [ - 8.000   -4.0  -17.0   x ]
           [    x         x     x     x ]
```

```
B      =  [ 1.0   0.0   0.0  -359.0 ]
           [ 0.0   1.0   0.0   281.0 ]
           [ 0.0   0.0   1.0   85.0 ]
           [  x     x     x     x ]
```

LEQTLF-2

CALL LEQTLF (A,M,N,IA,B,IDGT,WKAREA,IER)

.
.
.

END

Output:

IER = 0

A =
$$\begin{bmatrix} -8.0 & -4.0 & -17.0 & x \\ 3.0 & 2.0 & -6.0 & x \\ -4.1250 & -0.25 & 0.375 & x \\ x & x & x & x \end{bmatrix}$$

B =
$$\begin{bmatrix} -9.6666 & -2.6666 & -32.0 & 1.0 \\ 8.0 & 2.5 & 25.5 & -2.0 \\ 2.6666 & .6666 & 9.0 & -5.0 \\ x & x & x & x \end{bmatrix} \begin{matrix} Y_1 \\ Y_2 \\ Y_3 \\ \end{matrix}$$

Note: x indicates elements not used by LEQTLF.

[illegible]

NOV 17, 1980
1910215 80J 4095 6145161,
OUFUE: 3
ORIGINATING UNIT: 28

PRIORITY: 80
USFRCODE: PMCS00G1.
CHARGECODE: 000165.
SYSTEM/FCSTRAN ON PACK.
USFRCODE: PMCS00G1. CHARGECODE: 000165.
3.168 SEC CPU, 3.725 SECS IO
30 CARDS READ, 107 LINES PRINTED. 0 BACKUP TAPE LINES PRINTED.
MEM.ALLOC: 0.137 MBY. CUSTD \$12.26
ELAPSED TIME: 00:00:26

19:02:42 EOT 4132

\$63.40

19:02:46 EOT 4133 (PMCS00G1) 4095/LEOT UN PACK.
USFRCODE: PMCS00G1. CHARGECODE: 000165.
0.759 SEC CPU, 0.425 SECS IO
34 CARDS READ, 19 LINES PRINTED. 0 BACKUP TAPE LINES PRINTED.
MEM.ALLOC: 0.014 MBY. CUSTD \$1.26
ELAPSED TIME: 00:00:03

\$16.57

19:02:49 BILL 4095
EOT 4095 USFRCODE: PMCS00G1. \$0.02
61.5181.
USFRCODE: PMCS00G1. CHARGECODE: 000165.
0.335 SEC CPU, 0.395 SECS IO
MEM.ALLOC: 0.011 MBY. CUSTD \$0.95
ELAPSED TIME: 00:00:33

\$3.91

VALDR 00 JOB \$0.02 (RSTAM 93 PROCESSAMENTOS)


```
DIMENSION A(16,16),B(16),WKAFEA(32)
READ(5,10)((A(I,J),J=1,16),I=1,16)
```

N=16
IA=16

20 FERNAT(1X,10X,124,5DLUCAD DO SISTEMA)

```
30 FCPRMAT(1X,15X,7H,CASO A)  
WRITE(6,40)(B(I),I=1,14)
```

WRI TE(6,50)

50 FORMAT(IX,24H,PARABENS PELA FORMATURA)
STOP
END

```

C 002:002B:2
C 002:002B:2
C 002:002B:2
C 002:002F:2
C 002:002F:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003B:2
C 002:003F:2
C 002:003F:2
C 002:0040:1
SEGMENT 002 IS 004 LONG

```

FORMAT SEGMENT IS 007C LONG
START OF SEGMENT 005

WARNING:THE SUBROUTINE "LEQ1IF" WAS NOT FOUND

```

NO ERRORS DETECTED.  NUMBER OF CARDS = 30.
COMPILATION TIME = 10 SECONDS ELAPSED.  1.1 SECONDS PROCESSING.
02 STACK SIZE = 10 WORDS.  FILESIZE = 140 WORDS.  ESTIMATED CORE
TOTAL PROGRAM CODE = 152 WORDS.  ARRAY STORAGE = 304 WORDS.
NUMBER OF PROGRAM SEGMENTS = 6.  NUMBER OF LINK SEGMENTS = 41.
PROGRAM CODE FILE = (PWS00G1)LEOTF ON PACK.
COMPILER COMPILED ON 09/07/79 (FORTRAN ON PACK).

```

LEOTIF
=====

BIND=FROM BIP/IMSL/1,BIB/IMSL/2,BIF/IMSL/3
BEGIN BINDING LEOTIF OF .DUT FROM BIP/IMSL/2

LEOTIF (02,0002) CHANGED TO (02,0006)
LEOTIF (02,0005) CHANGED TO (02,000A)
LEOTIF (02,0004) CHANGED TO (02,000B)
LEOTIF (02,0003) CHANGED TO (02,000C)
LEOTIF (01,0002) CHANGED TO (01,0007) = 03 000004B00007
LEOTIF (01,0003) CHANGED TO (01,0009) = 05 080000140009

END OF BINDING LEOTIF
BEGIN BINDING UERTIF OF

UERTIF (02,0002) CHANGED TO (02,000A)
UERTIF (02,0009) CHANGED TO (02,000D)
UERTIF (02,0004) CHANGED TO (02,000F)
UERTIF (02,0005) CHANGED TO (02,0010)
UERTIF (02,0006) CHANGED TO (02,0011)
UERTIF (02,0007) CHANGED TO (02,0012)
UERTIF (02,0008) CHANGED TO (02,0013)
UERTIF (01,0002) CHANGED TO (01,000A) = 03 000000C0000D
UERTIF (01,0003) CHANGED TO (01,000B) = 05 07000000000A
UERTIF (01,0006) CHANGED TO (01,000C) = 05 0800004D40014

END OF BINDING UERTIF
BEGIN BINDING LUFLMF OF

LUFLMF (02,0002) CHANGED TO (02,000B)
LUFLMF (01,0002) CHANGED TO (01,000D) = 03 00000420001A
LUFLMF (01,0003) CHANGED TO (01,000F)
LUFLMF (01,0004) CHANGED TO (01,0010)
LUFLMF (01,0005) CHANGED TO (01,0011)
LUFLMF (01,0006) CHANGED TO (01,0012)
LUFLMF (01,0007) CHANGED TO (01,0013)
LUFLMF (01,0008) CHANGED TO (01,0014)
LUFLMF (01,0009) CHANGED TO (01,0015)
LUFLMF (01,000A) CHANGED TO (01,0016)
LUFLMF (01,000B) CHANGED TO (01,0017)
LUFLMF (01,000C) CHANGED TO (01,0018)
LUFLMF (01,000D) CHANGED TO (01,0019)
LUFLMF (01,000E) CHANGED TO (01,001A)
LUFLMF (01,000F) CHANGED TO (01,001B) = 03 000000500024
LUFLMF (01,000A) CHANGED TO (01,001C) = 05 0700000000011
LUFLMF (01,000B) CHANGED TO (01,001D) = 05 08000014002C

END OF BINDING LUFLMF
BEGIN BINDING LUDATF OF

LUDATF (02,0002) CHANGED TO (02,000C)
LUDATF (02,0003) CHANGED TO (02,000A)
LUDATF (02,0004) CHANGED TO (02,0014)
LUDATF (02,0005) CHANGED TO (02,0015)
LUDATF (02,0006) CHANGED TO (02,0016)
LUDATF (02,0007) CHANGED TO (02,0017)
LUDATF (02,0008) CHANGED TO (02,0018)
LUDATF (02,0009) CHANGED TO (02,0019)
LUDATF (02,000A) CHANGED TO (02,001A)
LUDATF (02,000B) CHANGED TO (02,001B) = 03 000000500024
LUDATF (02,000C) CHANGED TO (02,001C) = 05 0700000000011
LUDATF (02,000D) CHANGED TO (02,001D) = 05 08000014002C

END OF BINDING LUDATF
BEGIN BINDING UGETID OF

UGETID (02,0002) CHANGED TO (02,000D)
UGETID (02,0003) CHANGED TO (02,0019)
UGETID (02,0004) CHANGED TO (02,001A)
UGETID (01,0002) CHANGED TO (01,0011) = 03 00000110002F
UGETID (01,0003) CHANGED TO (01,0012)
UGETID (01,0004) CHANGED TO (01,0013)
UGETID (01,0005) CHANGED TO (01,0014)
UGETID (01,0006) CHANGED TO (01,0015)
UGETID (01,0007) CHANGED TO (01,0016)
UGETID (01,0008) CHANGED TO (01,0017)
UGETID (01,0009) CHANGED TO (01,0018)
UGETID (01,000A) CHANGED TO (01,0019)
UGETID (01,000B) CHANGED TO (01,001A)
UGETID (01,000C) CHANGED TO (01,001B) = 03 000000500024
UGETID (01,000D) CHANGED TO (01,001C) = 05 0700000000011
UGETID (01,000E) CHANGED TO (01,001D) = 05 08000014002C

END OF BINDING UGETID
NUMBER OF ERRORS DETECTED = 0.

HOST FILE = POST
SEGMENT DICTIONARY LENGTH = 18. GLOBAL STACK SIZE = 27. STACK ESTIMATE = 512.
CORE ESTIMATE = 618 WORDS. CODE FILE LENGTH = 68 DISK SEGMENTS.
BINDING TIME = 12 SECONDS ELAPSED, 2.83 SECONDS PROCESSOR, 3.56 SECONDS I/O.

RESOLUCAO DO SISTEMA

E=	0.0000000
F=	0.0506755
G=	-0.0506755
H=	0.0000000
I=	0.2202332
J=	0.0299159
K=	0.5000000
L=	-0.0171634
M=	-0.0151012
N=	-0.0004598
O=	0.0171634
P=	0.0151012
Q=	0.0000000
R=	0.2202332
S=	0.0299159
T=	-0.0004598
U=	0.0171634
V=	-0.0151012
W=	-0.0004598
X=	0.0171634
Y=	0.0151012
Z=	0.0000000

PARABENS PELA FORMATURA

^SOLUCAO DO SISTEMA

E=	0.1695595
F=	0.0299219
G=	-0.0171054
H=	0.0355725
I=	0.0000000
J=	0.0506755
K=	0.4700724
L=	-0.0506755
M=	0.0000000
N=	0.0171034
O=	0.0146416
P=	0.0171034
Q=	0.0171034
R=	0.0171034
S=	0.0151012
T=	0.2202332
U=	0.2197666
V=	0.0299199
Z=	

^PARABENS PELA FORMATURA

*SOLUCAO DO SISTEMA
*CASO C
E= 0.1695595
F= 0.1841963
G= 0.1841963
H= 0.0000000
I= 0.1695595
J= 0.0171054
K= 0.5000000
L= 0.0299219
M= 0.0335725
N= 0.0506755
O= 0.0299219
P= 0.0335725
Q= 0.0506755
R= 0.0299219
S= 0.0335725
T= 0.0000000
U= 0.0000000
V= 0.0000000
W= 0.0171054
X= 0.0000000
Y= 0.0000000
Z= 0.0506755
*PARABENS PELA FORMATURA