UNIVERSIDADE DE SÃO PAULO
FACULDADE DE ECONOMIA, ADMINISTRAÇÃO E CONTABILIDADE E ATUÁRIA
DEPARTAMENTO DE ECONOMIA

ESTIMATING A RISK-ADJUSTED CASH FLOW FOR FIXED INCOME SECURITIES

PAULO HENRIQUE BRUGOGNOLLE DE SOUZA
Número USP: 11764793

Academic Supervisor: Prof. Dr. Rodrigo De Losso da Silveira Bueno

São Paulo
2024

PAULO HENRIQUE BRUGOGNGOLLE DE SOUZA

ESTIMATING A RISK-ADJUSTED CASH FLOW FOR FIXED INCOME SECURITIES

Monograph presented to the Department of Economics of the University of São Paulo, as a partial requirement for obtaining a bachelor's degree in Economics.

Academic Supervisor: Professor Rodrigo De Losso da Silveira Bueno

SÃO PAULO

2024

SPECIAL THANKS

To my mother, Janaina, for the care, nurture and dedication throughout my whole life, which molded me as a person and allowed me to get here.

To my father, Mário, for the support, trust and for challenging me to go further and reach new goals.

To my grandfather, Fausto, for the care, the motivation to become an economist and for always broadening my horizons.

To my grandparents, Iolanda and Norival, for the constant support and care.

To my brother, Matheus, for the care and admiration, that makes me pursue my best version every day.

To my entire family, for supporting me in my decisions and providing me with the foundations on which to build my path.

To my professors, for the dedication to Economics and for the transmission of their knowledge to the alumni of the University. Special thanks to Professor José Carlos Santos, whose “Fixed Income Markets” classes were essential for this monograph.

To my Academic Supervisor, Professor Rodrigo De Losso, for the support in this work and for the reflections on the matter.

To my classmates, for the many semesters of partnership.

To my first boss, Alexander Ian Carpenter, for the lessons in the period I worked in his team, as well as for the initial spark that culminated in the theme chosen for this monograph.

To my work colleagues, for the partnership and trust, which make me strive to deliver my best every day.

SUMMARY

RESUMO

ESTIMATING A RISK-ADJUSTED CASH FLOW FOR FIXED INCOME SECURITIES

Historicamente, diversos economistas propuseram métodos de decomposição do prêmio de risco teórico de um título de renda fixa em fatores específicos, elaborando conceitos fundamentais para a precificação desses ativos e dinâmicas de taxas de juros. Todavia, a discussão sobre como o risco de calote e as suas consequências afetam indicadores-chave para ativos de renda fixa é um tópico pouco endereçado. Dessa forma, esse trabalho propõe uma estrutura que pode ser utilizada para ajustar o fluxo de caixa esperado de um título de renda fixa pelo risco de calote, dada o risco esperado de calote em cada período e a expectativa de recuperação do principal caso ele ocorra. Adicionalmente, depois de apresentar essa estrutura, essa monografia também endereça as principais implicações que esse ajuste tem nas principais métricas, como TIR, Duração e Convexidade.

Palavras-Chave: Risco de Calote, Retorno esperado de títulos de renda fixa, *Yield* ajustado ao risco.

ABSTRACT

ESTIMATING A RISK-ADJUSTED CASH FLOW FOR FIXED INCOME SECURITIES

Historically, several researchers have tried to decompose a bond's theoretical or actual risk premium into specific factors, providing paramount theories about bond pricing and interest rate dynamics. However, to understand how default risk and its consequences affect key indicators for bond markets is an under discussed topic. In this way, this paper aims to propose a structure that may be used to adjust the expected cash flow of a bond for the default risk, given the expected default rate in each period and the expected recovery value in the event of default. In addition, after providing the proposed framework, this paper also discusses the main implications of the cash flow adjustment, evaluating its impact on key bond metrics, including Yield to Maturity, Duration, Convexity etc.

Key words: Default Risk, Bond Expected Return, Risk-Adjusted Yield to Maturity.

1 INTRODUCTION

After one of the most aggressive hiking cycles in decades by central banks all over the world, investors have become more interested in fixed income after a long period of a relative overlooking due to an environment of ultra-low interest rates. The universe of fixed income securities is vast, and with more attention given to this major asset class, investors must have tools to compare different investment opportunities.

Therefore, this paper’s main purpose is to provide a framework for investors to compare the expected returns of bonds with different default risks, adjusting the cash flow by that variable. As will be discussed further, this is a probability of default assigned for each period, which can be based on historical data, credit analysis, or any other proprietary method. After that adjustment, investors may be able to compare bonds “excluding” the default risk, making the risks more tangible and hopefully reducing information asymmetry in the fixed income market.

In order to clarify the aspects investors must be most attentive to considering the risks of fixed income investments, another intended contribution of this work is to evaluate the impact of each parameter (coupon rate, default probability and recovery value) on the expected return of the investment under the proposed framework.

To achieve these goals, readers will be first introduced to the Literature regarding the theme before presenting the proposed framework itself and its implications.

2 LITERATURE REVIEW

The literature on corporate bonds and global fixed income is vast, and arguably one of the most intriguing and debated topics is whether markets are efficient in pricing default risks. Economists have presented several ways of estimating default rates and expected returns for bonds, but considering the numerous variables involved in explaining defaults and returns on these assets, such as companies' financial position, market turmoil and monetary policy, it is not yet a consensus if the market is capable of pricing all these events in an over-the-counter market such as fixed income securities.

One of the first studies which aimed to tackle the reasons for corporate defaults and their pricing method in the financial market was released by Fischer (1959), arguing that the risk premium, defined as "(…) the difference between the market yield on a bond and the corresponding pure rate of interest ["risk free rate"]", can be estimated by the risk of default on the bond and its "marketability" – nowadays more commonly expressed as liquidity. The author then presented a linear regression in which the risk premium is decomposed as a function of (i) earnings variability, (ii) tracking period of solvency, (iii) equity/debt ratio and (iv) amount of bonds outstanding. Despite some rather arbitrary measures, such as considering exactly 9 years of earnings variability and the selected specific financial ratios rather than others, the resulting function is allegedly able to explain 81 percent of the total variance in the logarithm of the risk premium. This contribution is a further development of earlier approaches that only considered earnings variability, not considering liquidity.

Citing Fisher (1959), Altman (1989) mentions two other approaches to default and risk premiums besides modeling them through microfinance measures and statistical classifications. These alternative methods are: (i) comparing annual default rates and actual returns to bond holders over various time frames; and (ii) combining observed pricing and the inherent default risk premium with estimates of corporate bond defaults. Altman himself then proposed a new method for measuring defaults and losses, inspired by actuarial methods for assessing human mortality. This approach was developed due to the alleged inability of previous models to adjust annual default rates for different events that also alter the total amount of bonds outstanding besides defaults, such as maturities and the exercise of call options by the bond issuers. The author presents the “Marginal Mortality Rate (MMR)” concept, which is given by the ratio between the total value of defaulting debt in a specific year and the total value of bonds at the start of that year. The main methodological contribution, that utilizes the MMR concept, seems to be the “Cumulative Mortality Rate (CMR)”, which is the subtraction of the products of surviving populations (1-MMR) from one. As described by the author,

> The [cumulative] mortality rate is a value-weighted rate for the particular year after issuance, rather than an unweighted average. If we were simply to average each of the year-one rates, year-two rates, etc., our results would be susceptible to significant specific-year bias. (Altman, 1989)

Moving on, the author also compares the results obtained to the traditional method of only compounding average annual default rates, concluding that the previous method usually underestimates cumulative default probability. The author, however, emphasizes the previous method’s merits in assessing default probabilities for a given year’s issuance, but argues that the CMR method is more appropriate when observing multiple periods of time.

The second important contribution of this article is the study of recovery values of corporate bonds after default events for each rating notch (AAA, AA, A, BBB etc.). Even though issuers default and therefore interrupt the scheduled payment series, bondholders usually receive an amount equivalent to a percentage of the par/face value after the debt restructuring process. To estimate this, the author uses the market price of a bond after the default event as an approximation for the expected recovery value, assuming markets price it correctly. Using this method, as expected, the author finds that higher-rated bonds trade at significantly higher prices after default events compared to “junk” bonds, with the difference reaching 40% of the par value, as shown in the table below:

Table 1 – Average Price After Default by Original Bond Rating (ALTMAN, 1989)

Average Price After Default by Original Bond Rating (1971–1987)

Data are from S&P Bond Guides, 1971–1987.

Original Rating	Average Price After Default (Per $100)	Number of Observations
AAA	78.67	5
AA	79.29	13
A	45.90	19
BBB	45.30	22
BB	35.71	13
B	42.56	64
CCC	41.15	12
C	10.00	2
NR	31.18	23
Arithmetic Average or Total	44.58	173

Source: (ALTMAN, 1989)

Finally, the third but not least important contribution of this article is the evaluation of actual historical spreads obtained by investing in corporate bonds rather than the “risk-free rate”. The results are that, over a 10-year period, BB-rated bonds present the highest return after adjusting for default rates, but with all other corporate bond classes presenting better returns than “riskless” assets. Utilizing a more traditional economic approach based on efficient markets, the compensation for the risk incurred for investing in a corporate bond should be equal to the default risk itself, so the excess return over a “risk free rate” should tend to zero. The evidence presented by Altman (1989) may then be interpreted as a market inefficiency, given that there seems to exist a “real risk premium”, beyond the fair compensation for the risk incurred. In this way, historical data suggests that investors are more than compensated for the risk taken in investing in riskier bonds when considering only default rates, which are probably not the only factor that accounts for the “risk premium”. In that matter, Altman (1989) cites some other explanations for the return spreads between these classes of bonds and risk-free assets, such as variability of expected recovery values for lower-rated bonds, which would then lead investors to demand a higher premium, and liquidity risk, which is not included in the analysis.

A more recent article, authored by John Hull, Mirela Predescu, and Alan White (2005), gathers important previous works on default risk premiums and rating agencies' and banks' databases to argue that the market indeed offers a higher premium for corporate bonds than what would be demanded to compensate their default, liquidity and other disclosed risks.

Citing Altman (1989), the authors distinguish default probability into two categories: "real-world", derived from historical data; and "risk-neutral", intrinsically obtained from bond pricing. The first one is derived from average cumulative default rates published by Moody's and the second is approximated as the excessive return (yield), presented by a bond versus the risk-free rate divided by one minus the recovery rate, assumed to be 40%, a common assumption by market participants – and similar to data obtained in Altman (1989).

The results achieved by comparing "real-world" default rates to the ones priced in the market suggest that bondholders earn a significant higher risk premium. To support this thesis, the authors cite Fleming (2001), who presents an estimate for liquidity premium in treasury bonds, arguing that issuances with lower liquidity pay around 10 basis points (0.1%) more than those with higher liquidity. Even though this spread may be higher for corporate bonds, it hardly explains the "excessive" risk premium earned by investors, which varies from 38 bps to 264 basis points (bps) per year, referring to the calculations presented by the authors.

Another factor considered when attributing the excessive return are the "traders' expectations", which can be summarized as the uncertainty regarding future default rates, which could be much higher than long or short-term historical data, justifying the additional premium demanded by investors.

For the authors, however, the main driver for higher risk premiums is the "non-diversifiable" (or systematic) risk, which derives from the fact that bonds are not completely uncorrelated, and there are periods when average default rates are very high. This risk can hardly be totally diversified away within the asset class, leading bond traders to demand an extra return for bearing it. The authors discuss that lower-rated bonds are usually more affected by the general state of the economy, presenting even higher correlations with equity markets, for example.

The main result of the article is that indeed there are higher risk premiums than anticipated by asset pricing models, which try to reach an "efficient risk premium", and the article lists previous studies conducted by renowned researchers to support this thesis.

In conclusion, all these works have contributed to advancing the literature about default rates and risk premiums, presenting valid factors to be considered and important methods for assessing these economic variables. However, it has not come to my attention an available tool or method to compare bonds with different expectations of default rates, resulting in a "risk-adjusted yield". This way, the main objective of this paper is to present a framework for investors to achieve this by adjusting the cash flow of a bond for their expectations of default risk and recovery value.

3 DEVELOPMENT

In order to construct an adjusted cash flow that includes the uncertainty regarding its payment, first it is necessary to define the exogenous variables on which it will be based on. The fixed income guidebooks used as references for the Development and Implications sections are (FABOZZI, 2005) and (SANTOS e SILVA, 2017).

3.1 DEFAULT RISK

The default risk is defined as the chance of an issuer to incur in a debt restructuring, usually resulting in worse terms for investors, who may face an extension of the bond period and/or a reduction of the face value or interest rate. As in the Literature Review section, that risk is usually expressed as an annual probability, which represents the probability of an issuer, given its specific risks or credit rating, to restructure its debt. Another way of looking at default probabilities is to evaluate the Cumulative Default Rate, or the probability of an issuer to default on its debt throughout longer periods of time, such as 5 or 10 years. As mentioned, one of the most common methods for estimating this variable was proposed by Altman (1989), and consists of “a value-weighted rate for the particular year after issuance rather than an unweighted average”, mitigating the risk of specific-year bias in assessing the cumulative default probability.

Regardless of the method used to estimate it, the cumulative default probability is a pillar of the proposed method for adjusting the cash flow, as it can be used to establish a discount factor for each cash flow. In other words, the expected cash flow of a bond in a single period fully depends on the continuity of cash flows until that period, meaning that the probability of its payment depends not only on the probability of default in that period but also on the cumulative probability of default in previous periods.

3.2 DISCOUNT FACTOR

The discount factor is a consequence of the default risk, as each future cash flow must be adjusted by the default risk in each previous period before the current one. Assuming a simpler scenario in which the default probability is constant (d%) for each period, an estimate for the discount factor in each period is the probability of the issuer to have continued to pay its debt obligations $(1-d)$ exponentiated until the last period before the current one $(t-1)$. This calculation is analogous to that proposed by Predescu et. al (2005), as they exponentiate $(1-d)$ to estimate the probability of a bond issuer to survive for T years.

Table 2 – Default Probability vs Discount Factor

Period	Default Probability (%)	Discount Factor
1	d	$(1-d)^{(1-1)} = 1$
2	d	$(1-d)^{(2-1)} = (1-d)^1$
3	d	$(1-d)^{(3-1)} = (1-d)^2$
$\boldsymbol{t}$	d	$(1-d)^{t-1}$

In a more complex (and realistic) scenario, in which the estimate for the default probability is not constant, the calculation can be adapted by simply multiplying the probability of the issuer to continue to pay its debt obligations $(1-d)$ for each period prior to the current, or $(1-d_1)$ x $(1-d_2)$ x (...) x $(1-d_{t-1})$.

Nevertheless, for better comprehension of the examples, the simplification of a constant default probability for each period will be used.

3.3 RECOVERY VALUE

Another important exogenous variable for the risk-adjusted cash flow is the recovery value, defined as the expected percentage of the original face value recovered by investors if there is a renegotiation of the original terms of a specific bond (default). The recovery value can be estimated by the market price of a bond immediately after an event of default, as in Altman (1989), or as the value a bondholder could recover after the issuer has gone bankrupt, usually estimated as the liquidation value of the company after paying obligations ranked higher in payment priority to that specific debt security. One might also discuss the dynamics of the recovery value, perhaps considering it would be higher when closer to the maturity date, for example. Nevertheless, the proposed framework could be adapted to that case, by simply assuming a different expected recovery value for each period, but, for clarity, the recovery value will be considered as static from now on.

In the cash flow adjustment, the recovery value may have a dubious impact, since it can enhance the value of a cash flow in a specific period, as the recovery value is usually higher than the coupon payment. The cost of this is the interruption of the payment series itself, an event that negatively compensates for a specific higher cash flow in the default period.

For example, assuming a bond matures in 8 years, has a coupon rate of 6% per annum, and an expected recovery value of 60% of the face value, the difference between the cash flows in Scenario 1 (no Default) and 2 (Default on Year 5) is significant, even though the Scenario 2 cash flow in Year 5 may be higher:

Table 3 – IRR whether there is a Default or Not

Year	Scenario 1	Scenario 2
0	-100%	-100%
1	6%	6%
2	6%	6%
3	6%	6%
4	6%	6%
5	6%	60%
6	6%	–
7	6%	–
8	106%	–
IRR (%)	**6.00%**	**-3.96%**

3.4 MARKET PRICE AND COUPON RATE

These final two exogenous variables consist of (i) the market price of a bond (including accrued interest), usually expressed as a % of the face value, which generally has (ii) annual or semi-annual coupon payments that represent a % of the par value as well. The market price including accrued interest is also equivalent to the present value of all future cash flows, as scheduled, discounted by the yield-to-maturity, the Internal Rate of Return (IRR) of this bond. Usually, that last variable is the one players evaluate and trade the securities based on, given that it is the expected return of that investment. The Yield of a bond was also the main motivation of this work, especially the fact that it does not consider the uncertainty of the cash flow. Therefore, by creating a “Risk-Adjusted Cash Flow”, the main implication is estimating a “Risk-Adjusted Yield-to-Maturity” based on this.

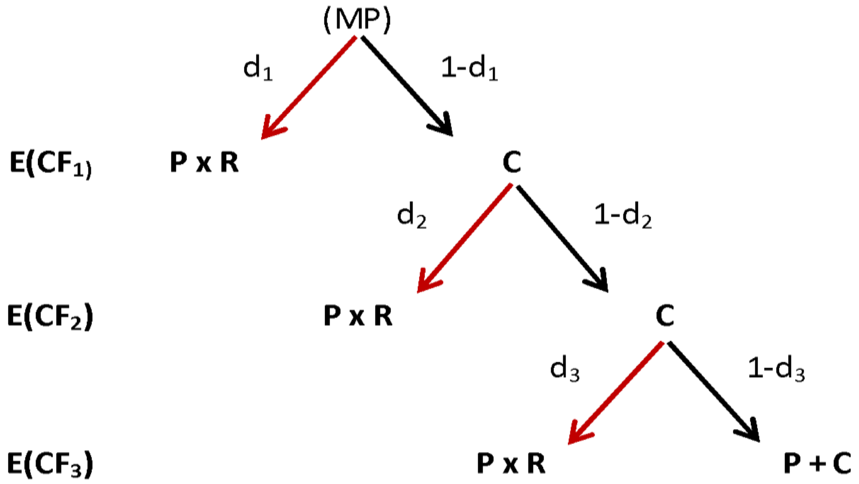
3.5 CONSTRUCTING THE RISK-ADJUSTED CASH FLOW

After defining the main variables that will be used in the construction of the adjusted cash flow, its concept must also be defined.

The idea of this structure is to represent the alternative scenarios for each bond event, whether it is a coupon payment or its redemption date. At each moment, there is a chance the issuer will continue to meet the scheduled payments, but also a chance it will renegotiate the terms leaving investors worse off, or, in other words, default on its bonds. Given those dual possibilities, the proposed structure aims to reflect both possibilities in an Expected Cash Flow for each period, or E(CFt). In addition, as explained in the discussion about the Discount Factor, the occurrence of each E(CFt) depends on the continuity of the payment series until the previous event, and that is why the E(CFt) should consider the Discount Factor.

The structure of the Adjusted Cash Flow would then be similar to the following:

Figure 1 – Proposed Structure of a Risk-Adjusted Cash Flow



MP = Market price of the bond including accrued interest (%)
$E(CF_t)$ = Expected cash flow for each coupon payment (%)
d_t = Default probability in each coupon payment (%)
R = Recovery value (% of par value)
C = Coupon payment (%)
P = Par value (%)

As explained earlier, a simpler version of this model can be obtained by assuming a constant default probability for each period, so that the discount factor for each period is similar to the proposed calculation, $(1\text{-d})^{(t-1)}$. In this simplified version, the Expected Cash Flow for each period can be obtained as shown in the following table:

Table 4 – Expected Cash Flows

Year	Discount Factor	Continuity	Default	E(CF)
0	−	−	−	$-MP$
1	$(1-d)^0 = 1$	$(1-d) \times C$	$d \times P \times R$	$1 \times [(1-d) \times C + d \times P \times R]$
2	$(1-d)^1$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^1 \times [(1-d) \times C + d \times P \times R]$
3	$(1-d)^2$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^{21} \times [(1-d) \times C + d \times P \times R]$
4	$(1-d)^3$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^3 \times [(1-d) \times C + d \times P \times R]$
5	$(1-d)^4$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^4 \times [(1-d) \times C + d \times P \times R]$
6	$(1-d)^5$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^5 \times [(1-d) \times C + d \times P \times R]$
7	$(1-d)^6$	$(1-d) \times C$	$d \times P \times R$	$(1-d)^6 \times [(1-d) \times C + d \times P \times R]$
8	$(1-d)^7$	$(1-d) \times (P+C)$	$d \times P \times R$	$(1-d)^7 \times [(1-d) \times (P+C) + d \times P \times R]$

The proposed structure emphasizes the importance of considering the prior events by including the discount factor in each expected cash flow calculation. The other side of the multiplication is a constant of $(1-d) \times C + d \times P \times R$, given that Default Rates are considered constant, as well as the Recovery Value.

Therefore, it is important to note that, by using an Expected cash flow with a binary option for each period, the probabilities of default in all prior periods are considered when calculating the probability for a specific event through the discount factor.

4 IMPLICATIONS

After providing the theoretical approach to the Risk-Adjusted Cash Flow, this next section aims to discuss its implications by presenting a practical case. This example will help better understand the changes and impacts this proposed model has on key variables, the (i) Yield-to-Maturity, (ii) Duration, (iii) Convexity and (iv) Term Structure. Additionally, a sensitivity analysis will help determine which variables (Coupon Rate, Recovery Value or Duration) the expected return on a bond is more sensitive to accounting for default risk.

4.1 THEORETICAL EXAMPLE

A corporate bond that matures in 8 years has an annual coupon rate of 5% and a Market Price (including accrued interest) of 85% of the face value. A credit analyst estimates that the annual default risk for this security is 2%, with an expected recovery value of 30% in the event of default. The traditional cash flow for this bond is as follows:

Table 5 – "Traditional" Cash Flow

Year	Cash Flow
0	-85%
1	5%
2	5%
3	5%
4	5%
5	5%
6	5%
7	5%
8	105%

As discussed earlier, this cash flow does not incorporate the uncertainty regarding the payments and risk of default. The estimated risk-adjusted cash flow is:

Table 6 – Risk-Adjusted Cash Flow Structure

Year	Discount Factor	Continuity	Default	E(CF)
0	–	–	–	-85%
1	$(1-2\%)^0$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$1\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
2	$(1-2\%)^1$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^1\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
3	$(1-2\%)^2$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^2\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
4	$(1-2\%)^3$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^3\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
5	$(1-2\%)^4$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^4\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
6	$(1-2\%)^5$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^5\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
7	$(1-2\%)^6$	$(1-2\%)\times 5\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^6\times[(1-2\%)\times 5\% + 2\%\times 100\%\times 30\%]$
8	$(1-2\%)^7$	$(1-2\%)\times 105\%$	$2\% \times 100\% \times 30\%$	$(1-2\%)^7\times[(1-2\%)\times 105\% + 2\%\times 100\%\times 30\%]$

Table 7 – Risk-Adjusted Cash Flow

Year	Discount Factor	Continuity	Default	E(CF)
0	–	–	–	-85.000%
1	1.000	4.900%	0.600%	5.500%
2	0.980	4.900%	0.600%	5.390%
3	0.960	4.900%	0.600%	5.282%
4	0.941	4.900%	0.600%	5.177%
5	0.922	4.900%	0.600%	5.073%
6	0.904	4.900%	0.600%	4.972%
7	0.886	4.900%	0.600%	4.872%
8	0.868	102.900%	0.600%	89.851%

The first result of this structure is the fact that the expected annual cash flows until year 6 are higher than the “traditional” cash flow, since the 2% chance of default in each period results in a recovery value equivalent to 30% of the face value, which is far higher than the coupon rate of 5%, with a 98% chance of occurring. The cost associated with this, as presented earlier, is the increasing chance of discontinuity in the series, or default, represented by an increasingly lower discount factor that reduces the expected cash flow for longer periods. This impact reaches its peak at the redemption date, where the E(CFt) equals to 89.851% of the Par Value, comparing to a cash flow of 105% when considering the unadjusted cash flow.

4.2 YIELD TO MATURITY

The yield to maturity of a bond based on a traditional cash flow is equivalent to the Internal Rate of Return, defined as the discount rate that equals the future cash flows to the Market Price, considering accrued interest. When evaluating a risk-adjusted cash flow, the proposal is similar, but now the cash flows are not certain since they incorporate the current and previous probabilities of default in each period, as expressed above.

In our proposed example, the difference between the traditional Yield-to-Maturity and the Risk-Adjusted Yield-to-Maturity is as follows:

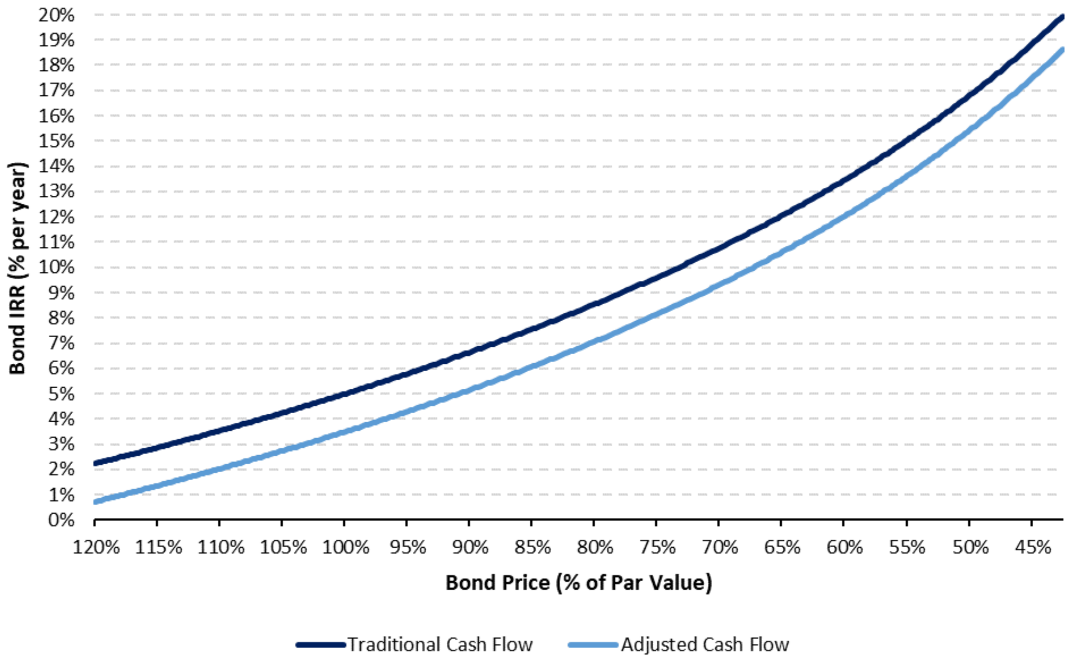
Table 8 – Yield to Maturity x Risk-Adjusted Yield to Maturity

Year	Cash Flow	Expected Cash Flow
0	-85%	-85.000%
1	5%	5.500%
2	5%	5.390%
3	5%	5.282%
4	5%	5.177%
5	5%	5.073%
6	5%	4.972%
7	5%	4.872%
8	105%	89.851%
YTM	7.568%	6.082%

The impact of an increasing cumulative default probability on the expected cash flow translates to the uncertainty of events that will happen far in the future. That fact, represented by an increasing discount factor based on default risk for each event, as seen, reduces the total cash flow of a bond. In this way, the Internal Rate of Return that equals the sum of the present value of those cash flows to the Market Price of the bond must be lower, reducing the expected return of the bond, as the logical conclusion would presume.

This may be considered the main implication of adjusting the cash flow of a bond for the default risk, and the implication investors may pay the most attention. After estimating a Risk-Adjusted Yield to Maturity, market agents may compare different assets and their investment attractiveness.

Graph 1 – Traditional and Adjusted Bond Yield x Price



An important consideration, which will be further explained in the "Statement of Limitations" section, is that this adjustment does not consider the variance of returns. For example, if two different bonds with similar characteristics present the same risk-adjusted yield to maturity, a rational investor should choose the one with lower default risk, given that, in most cases, it should deliver a return closer to the risk-adjusted yield to maturity than the riskier one. In other words, the lower return variability of the first bond is a desirable characteristic when facing a situation where the bonds present a similar risk-adjusted yield to maturity.

4.3 DURATION

After evaluating the impacts of the proposed model on the bond's yield, another important effect this framework has is on the bond's duration. Given that estimated cash flows present an irregular pattern, with those closer to the purchase date having a greater weight on the sum of cash flows due to the discount factor mechanism, the Macaulay and Modified Durations of a bond are lower with the proposed framework in comparison to a traditional cash flow. Another reason for that is the peculiarity of expected cash flows being higher in closer periods, as discussed previously in this section. The table below presents the unadjusted and adjusted cash flow duration metrics in order to evaluate the impact:

Table 9 – Traditional and Adjusted Weight on Duration

Year	Cash Flow	Weight on Duration	Expected Cash Flow	Weight on Duration
0	-85%		-85.000%	
1	5%	0.055	5.500%	0.061
2	5%	0.102	5.390%	0.113
3	5%	0.142	5.282%	0.156
4	5%	0.176	5.177%	0.192
5	5%	0.204	5.073%	0.222
6	5%	0.228	4.972%	0.246
7	5%	0.247	4.872%	0.265
8	105%	5.513	89.851%	5.273
Macaulay Duration		6.666		6.529
Modified Duration		6.197		6.155

The apparent reduction of the bond duration must be interpreted correctly: instead of presenting less risk, a conclusion one might draw from this information, it simply expresses a lower expected cash flow in the future periods, changing the relative weight of each cash flow in the series, with a greater weight carried by those closest to the observation date and a lower one to those in the future, given that their absolute value is reduced by the discount factor. In other words, the probability of default in each period results in an increasingly lower expected cash flow for each period, given that the cumulative default probability grows. In this way and as mentioned before, the main impact on the structure of the cash flow occurs on the redemption date, as it is usually the largest cash payment and the cumulative default probability is at its highest, which means the discount factor is the lowest in the series.

The following table shows how the cumulative cash flow is reduced by the increasingly higher cash flow adjustments, but also how the weight of the cash flows before the redemption period is shifted, reducing the bond duration.

Table 10 – Traditional and Adjusted Cumulative Cash Flow

Year	Cash Flow	Cumulative CF	Expected Cash Flow	Cumulative E(CF)
1	5.000%	5.000%	5.500%	5.500%
2	5.000%	10.000%	5.390%	10.890%
3	5.000%	15.000%	5.282%	16.172%
4	5.000%	20.000%	5.177%	21.349%
5	5.000%	25.000%	5.073%	26.422%
6	5.000%	30.000%	4.972%	31.393%
7	5.000%	35.000%	4.872%	36.265%
8	105.000%	140.000%	89.851%	126.116%

In this example, the combination of a 2% default probability in each period with an expected recovery value of 30% resulted in a higher expectation of cumulative cash flow for each period until the redemption date, which, as discussed, has the most meaningful impact on the payment series.

Another interesting interpretation of the lower duration after the proposed adjustment is that, as the expectation of future cash flows becomes lower, the correlation of the series with interest rate changes decreases. To illustrate this point, let us compare the expected cash flow in period 8. Using the “traditional” approach, the cash flow on period 8 is 105%, so a change of 1% in interest rates should impact the present value of that cash flow, which represents a significant part of the bond price. Alternatively, the expected cash flow in period 8 using the proposed approach is 89.851%. The same change of 1% in interest rates should, therefore, have less impact on the present value of that cash flow, leading to a lower change on the bond price. This interpretation also relates to the next topic.

4.4 CONVEXITY

Regarding the bond convexity, which is the second derivative of the bond price with respect to its IRR, adjusting the cash flow for default probability has an ambiguous effect. Even though it increases the coupon payment variance by reducing the concentration of the redemption payment's weight on the present value of cash flows – a change that would be accretive to convexity –, this adjustment, as presented above, also reduces the duration, which significantly impacts convexity. Thus, the net effect on convexity is the sum of these two opposing effects, which may be positive or negative depending on the case. In our example, the effect is at follows:

Table 9 – Traditional and Adjusted Weight on Convexity

Year	Cash Flow	Weight on Convexity	Expected Cash Flow	Weight on Convexity
0	-85%		-85.000%	
1	5%	0.109	5.500%	0.122
2	5%	0.305	5.390%	0.338
3	5%	0.567	5.282%	0.625
4	5%	0.879	5.177%	0.962
5	5%	1.225	5.073%	1.333
6	5%	1.595	4.972%	1.724
7	5%	1.977	4.872%	2.123
8	105%	49.620	89.851%	47.457
Sum of Weights		56.277		54.683
$\frac{1}{(1+IRR)^2}$		0.864		0.889
Convexity		48.64		48.59

In the presented example, the net effect on convexity is almost neutral, suggesting a counterbalance of the opposite effects of duration reduction and increased weight distribution mentioned above.

4.5 TERM STRUCTURE

Lastly, it is important to evaluate how adjusting the cash flow for default probability impacts the term structure of the curve. For this, bonds with multiple maturity dates are needed. Suppose a company has issued 3 bonds, all with a coupon rate of 5% per year, with one maturing in 1 year, another in 2 years and the third in 3 years. The market prices for each bond are 100%, 95% and 90%, respectively. The estimated default probability in each period is 2%, with an expected recovery value of 30%. The adjusted cash flows are as follows:

Table 10 – Cash Flows vs Expected Cash Flow

Year	Bond 1	E(CFt)	Bond 2	E(CFt)	Bond 3	E(CFt)
0	-100%	-100%	-95%	-95%	-90%	-90%
1	105%	103.50%	5%	5.50%	5%	5.50%
2			105%	101.43%	5%	5.39%
3					105%	99.40%

By utilizing the bootstrapping method to estimate the zero-coupon yield curve for this issuer, we can calculate the term structure of both cash flow structures – the “traditional” one and the expected cash flow adjusted for default risk:

Table 11 – Spot Rates vs Term Rates

Year	Spot Rates CF	Term Rates CF	Spot Rates E(CF)	Term Rates E(CF)	CF / E(CF) Term Rate Differential
1	5.00%	5.0%	3.50%	3.50%	1.45%
2	7.80%	10.82%	6.26%	9.27%	1.42%
3	8.95%	11.18%	7.41%	9.63%	1.41%

From this example, it is observable that the main impact of cash flow adjustment is a reduction in term rates, proportionate to the reduction in Yield to Maturity. The relative impact, however, is not material. It is important to note that utilizing a variable default probability and recovery value could alter this result.

4.6 SENSITIVITY ANALYSIS

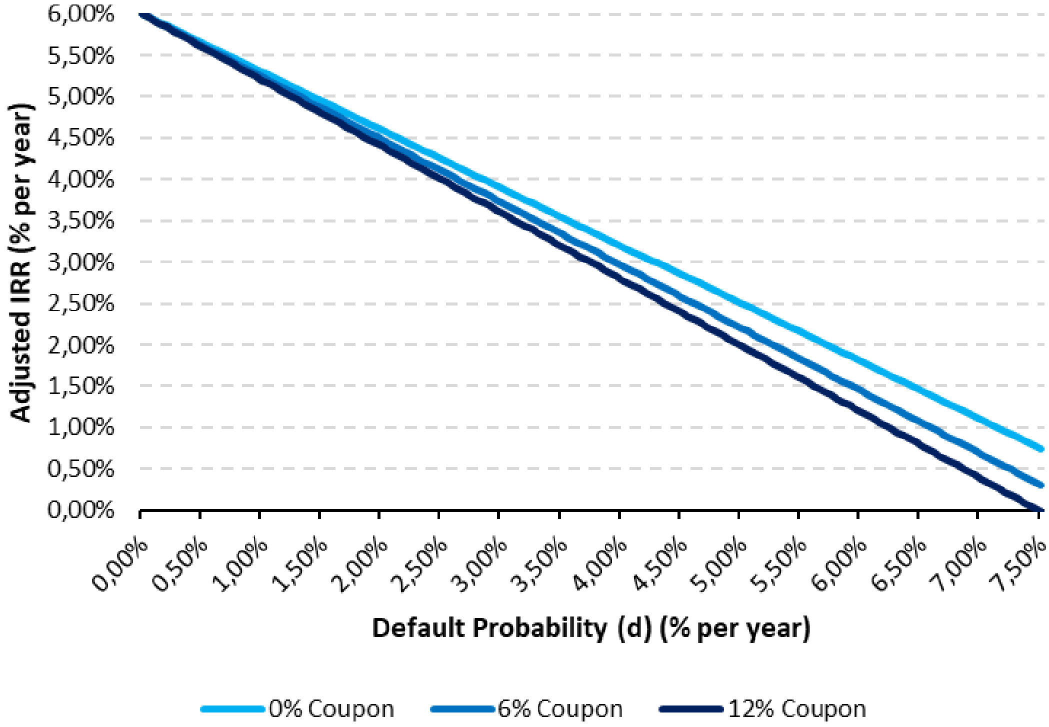
After evaluating the implications of the cash flow adjustment on important metrics for bonds through a specific example, the following section aims to evaluate which characteristics of a bond result in less reduction of the expected return (risk-adjusted yield to maturity). Presumably, after reaching this conclusion, investors should be able to prioritize certain aspects in the bond selection process to mitigate the negative impacts of default risk.

The analysis consists of comparing bonds with similar characteristics except for the one being evaluated (Coupon Rate and Recovery Value) and the default probability, which is the determining factor for the IRR reduction. First, the comparison between bonds with different coupon reveals an interesting outcome:

Table 12 – Parameters of the Coupon Rate Sensitivity Analysis

Parameters	Bond 1	Bond 2	Bond 3
Coupon Rate	0%	6%	12%
Recovery Value	30%	30%	30%
Initial IRR ($d = 0\%$)	6.00%	6.00%	6.00%
Initial Price	74.73%	100.00%	125.27%

Graph 2 – Adjusted IRR vs Default Probability for Bonds with Different Coupon Rates

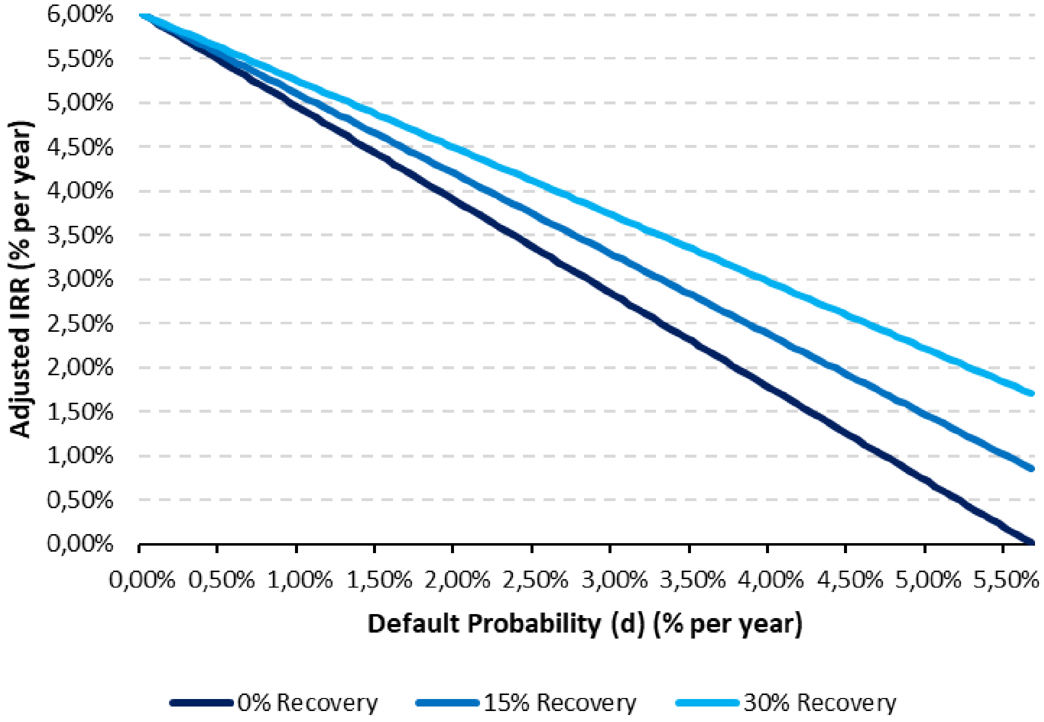


The results are relatively surprising at first glance, considering that the bond with the highest coupon rate experienced the sharpest decline in the adjusted IRR with an increase in default probability. This conclusion, however, derives from the fact that the three theoretical bonds had the same IRR in the first place, which means the price paid for the 12% coupon bond had to be significantly higher compared to the other two bonds. This conclusion implies that paying a lower price for the bond mitigates the risk of permanent loss better than a high coupon rate. It is important to note that this analysis considers the complete cash flow over all periods – a higher coupon rates anticipates a fraction of the expected future cash flows, but at the cost of paying a higher price initially.

Table 13 – Parameters of the Recovery Value Sensitivity Analysis

Parameters	Bond 1	Bond 2	Bond 3
Coupon Rate	6%	6%	6%
Recovery Value	0%	15%	30%
Initial IRR ($d = 0\%$)	6.00%	6.00%	6.00%
Initial Price	100.00%	100.00%	100.00%

Graph 3 – Adjusted IRR vs Default Probability for Bonds with Different Recovery Values



As observed, the relation between recovery value and adjusted IRR is inversely proportional as default probability increases. In the limit, a bond with an expected recovery value of 100% would deliver the same adjusted IRR regardless of default risk. However, in real markets, it is almost impossible to attribute this expected recovery value to “clean” bonds, which are not backed by assets as guarantees. Even having assets as guarantees is usually sufficient to expect a recovery value of 100% in most cases.

5 STATEMENT OF LIMITATIONS

After discussing the main objectives of the research and its proposed methodology, it is also important to highlight its limitations.

Although default rates and recovery values may be cited and used to support the argument that the market may not be completely efficient in pricing them, this research does not intend to propose a method for calculating them. Therefore, it is important to reiterate that both variables are exogenous in the proposed approach and should be estimated or obtained by those who wish to use the proposed cash flow adjustment. Nevertheless, the accuracy of the proposed framework also depends on the validity of these variables.

A limitation that may arise is the issue of default risk variance. As mentioned in the Literature review, default rates may vary significantly in different economic scenarios, and this variability may not be incorporated into the adjustment proposed by this research. It is up to the final users to run stress tests to evaluate the impact of variations in their inputted default risk in each case. In other words, the IRR calculation results in a single number, which does not include “tail” risks when default rates vary significantly from those assumed by investors. This may justify the higher risk premiums offered by lower-rated bonds, for example.

Additionally, there is the matter of the expected return variance. Two bonds may present an equal Expected Return as measured by the risk-adjusted yield to maturity, but a rational investor should always choose the one with lower risk, as evaluated by themselves.

The proposed method also considers discrete periods of time (quarters, semesters and years), meaning that default rates must be inputted in relatively large periods, instead of considering the continuous probability of the issuer defaulting on its bonds or the probability of a default occurring between coupon payments.

Finally, other important factors for bond pricing, such as liquidity, are not included in the adjustment, resulting in a simplified model for the sake final investors’ usability.

6 CONCLUSION

As discussed in the Literature Review section, although several theories have been created aiming to determine the factors explaining risk premium, some studies show a “real” risk premium has been historically seen in some fixed income securities, suggesting there might be some level of inefficiency in this market, as would be expected in such a complex and vast universe. Therefore, in the Development section, a framework is proposed to achieve a “risk-adjusted cash flow”, which represents an expected cash flow that includes the level of default risk investors project for that security. The Implications section, in addition, evaluates the impacts of that adjustment on the bond yield, duration, convexity and, through sensitivity analysis, aims to provide a perception of which variables are most important to maximize the expected return of a bond considering the default risk.

Finally, the Statement of Limitations is intended to delineate the scope of this work, which does not intend to calculate default risk, recovery value, or other variables. It provides only an expected cash flow, which will differ from reality for each fixed income security, as it will follow only the path of Continuity or Default.

In addition, possible advances on this work could incorporate the return variance of bonds, given that while the uncertainty brought by default risk is incorporated into the model, it provides only a specific value of expected return, which does not present all possible scenarios of return for that security. For example, a bond may have a risk-adjusted yield to maturity of 7% per annum with a range varying from -5% per annum to 35% per annum. These ranges are incorporated into the result but given less focus.

Nevertheless, after (i) presenting the academic discussion regarding default risks and risk premium, (ii) introducing a framework for adjusting fixed income securities cash flows for default risk, and (iii) modelling its impacts on the main variables related to bonds, this paper is concluded having addressed its main objective.

REFERENCES

ALTMAN, E. I. Measuring corporate bond mortality and performance. **The journal of finance**, v. 44, n. 4, p. 909–922, 1989.

FABOZZI, F. J. **The handbook of fixed income securities**. 7. ed. Nova Iorque, NY, USA: McGraw-Hill Professional, 2005.

FISHER, L. Determinants of risk premiums on corporate bonds. **The journal of political economy**, v. 67, n. 3, p. 217–237, 1959.

FLEMING, M. J. Measuring treasury market liquidity. **SSRN Electronic Journal**, 2001.

HULL, J.; PREDESCU, M.; WHITE, A. Bond prices, default probabilities and risk premiums. **The journal of credit risk**, v. 1, n. 2, p. 53–60, 2005.

SANTOS, José Carlos de Souza e SILVA, Marcos Eugênio da. **Derivativos e renda fixa: teoria e aplicações ao mercado brasileiro**. São Paulo: Atlas, 2017.