ALEX TETSU MIYAGI

Analysis of 5 bottlenecks for Lithium and Boron extraction from offshore platforms, considering environmental and financial factors

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Graduation Work presented to the *Escola Politécnica* – University of Sao Paulo for attaining the Bachelor's Degree in Production Engineering

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To family and friends.

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“[…] success is to be measured not so much by the position that one has reached in life as by the obstacles which he has overcome while trying to succeed”

(Booker T. Washington)

ABSTRACT

Lithium consumption has significantly increased in recent years and is expected to reach 415 thousand tons by 2050. Also, lithium-ion batteries currently dominate the global lithium market, accounting for 65% of the demand and are projected to reach 90% by 2025. Lithium can be extracted from different sources, including mineral ores, electronic waste, salt-lake brines, seawater, and geothermal brines. However, new sources are needed to meet the growing demand. The source analyzed in this study is produced water from offshore platforms. Produced water is extracted from oil reservoirs along with the oil and constitutes a significant portion of oil and gas wastewater. Recovery of lithium from produced water could be combined with removing boron present, a chemical with specific environmental restrictions regarding its concentration in water for disposal or human consumption.

Using concepts such as Monte Carlo Simulation and confidence intervals, this study proposes analyzing 5 bottlenecks for direct removing lithium and boron from produced water, considering environmental and economic risks. From the obtained results, it was possible to broaden the knowledge on extracting the chemicals from produced water and quantify the viability of each bottleneck. Further studies are considering different hypotheses for each bottleneck and an integrated system to overview every variable in this system simultaneously.

Keywords: Produced water. Lithium. Boron. Monte Carlo simulation. Economic risk.

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1. INTRODUCTION

This topic presents the motivation and objective for this project's development. Also, the structure of the study is divided and organized for synthetizing the study.

1.1. Motivation

Lithium is widely available on the Earth's surface, with a concentration ranging from 20 to 70 parts per million (ppm), making it the 25th most abundant element (MESHRAM, PANDEY, & MANKHAND, 2014; L. ZHANG et al., 2018). Its consumption has nearly tripled over the past decade and is projected to reach 415 thousand tons by 2050 due to initiatives to minimize negative effects of climate change. Lithium-ion batteries (LIBs) currently dominate the global lithium market, representing 65% of the demand and potentially reaching 90% by 2025, with LIBs alone requiring 174 thousand tons of lithium (TABELIN et al., 2021). In addition to batteries, lithium is used in various applications such as glass, ceramics, greases, lubricants, metallurgy, air conditioning, and aluminum production (MESHRAM, PANDEY, & MANKHAND, 2014). While lithium can be extracted from different sources, including mineral ores, electronic waste, salt-lake brines, seawater, and geothermal brines, new sources are needed to meet the growing demand.

On the other hand, boron is a micronutrient essential for plant growth and has clinical significance. While boron deficiency can be detrimental, excess boron is hazardous and more commonly encountered. Boron exists in nature primarily as boric acid, borates, or boron-silicates. The concentration of boron in the ocean, its primary source, ranges from 0.5 to 9.6 mg/L (HILAL, KIM, & SOMERFIELD, 2011). As boron is abundant in produced water (PW) from the oil and gas industry, its presence can cause environmental contamination, necessitating its separation and recovery for industrial use to comply with environmental regulations when disposing wastewater in the ocean.

PW is the water extracted from oil reservoirs along with the oil and constitutes a significant portion of oil and gas wastewater (FAKHRU'L-RAZI et al., 2009). It contains various organic and inorganic compounds such as dissolved oil, grease, heavy metals, salts, waxes, dissolved oxygen, gases, and formation solids (IGUNNU & CHEN, 2014; JIMÉNEZ et al., 2018). PW volume exceeds that of oil and other hydrocarbons in the extraction process, and its disposal is strictly regulated, mainly requiring disposal back into the ocean, which impacts the economic feasibility of oil fields (BAGHERI, ROSHANDEL, & SHAYEGAN, 2018). Although lithium concentration in PW is low, utilizing PW for lithium and boron extraction can benefit oil producers economically (KUMAR et al., 2019). Therefore, this study proposes the utilization of PW to extract lithium and boron, which can positively impact the economic feasibility of offshore platforms and align with the safe disposal policies for chemicals in the environment.

This study was performed with a research team to determine the risks associated with lithium and boron direct removal technologies. It served as support to deepen the knowledge of bottlenecks in technology's success. Through expert elicitations and a qualitative mind map deployment, the team reached five bottlenecks, which allowed the development of this study.

1.2. **Objectives**

This study analyzes different bottlenecks for removing lithium and boron from produced water and verifying them from environmental and economic factors. The environmental risk is related to the maximum limit of boron release in the seawater (due to the hazardous of the chemical) and other reuse water use, and the financial risks involve analyzing the feasibility of lithium commerce, mainly from produced water, so that it can be viable financially for an offshore platform to extract these chemicals.

Different methodologies and variables are available for study when considering the direct lithium and boron removal (DLBR) from produced water. This considerable amount of information must be analyzed, filtered, and selected to provide a good quality review of which kind of bottlenecks can be accepted for environmental and financial risk; while also being able to make a critical analysis through hypothesis, statistical evidence, and software manipulation.

1.3. Structure of the Graduation Work

To provide the study proposed, the organization of the graduation work will be as follows:

Chapter 2: How lithium and boron can be extracted from produced water and how the technologies and variables were selected through mind maps and systematic review. This chapter also presents the adopted analysis techniques: Monte Carlo simulations, distribution approaches and confidence intervals.

Chapter 3: The first bottleneck relates to environmental regulation and boron extraction. Boron is an essential chemical for treating osteoarthritis and building fortified bones, and it can also be used for plant growth or seed formation. However, when in high concentrations, boron can also be considered hazardous, influencing animals and the environment. Therefore, it is necessary to make an analysis for this bottleneck and analyze different countries (environmental regulations) and produced water (concentration of boron) for this to be viable.

Chapter 4: The next bottleneck connects ion boron concentration and geothermal position. Different geological locations can have different concentrations of ions in the produced water, interfering with the final boron concentration disposed of in seawater. Therefore, an analysis should be made to avoid environmental penalties and taxes.

Chapter 5: This chapter will explain the third bottleneck for the extraction of boron and lithium, the market price. Considering lithium prices in the market, it is necessary to see how financially viable the technologies can be.

Chapter 6: The fourth bottleneck concerns the efficiency of available technologies that can be used for lithium extraction in produced water. Due to similar characteristics between lithium and other ions in produced water, an overall analysis of how the efficiency of systems influences economic feasibility is necessary.

Chapter 7: The final bottleneck is connected to the microemulsion and system setup. For extraction of lithium and boron from the produced water, it is necessary to do electrochemical and membrane processes, respectively. Nevertheless, microemulsion can interfere with the system. Thus, an analysis should be made to verify the advantages and disadvantages microemulsion can provide in the project.

Chapter 8: Overview and conclusions of the study.

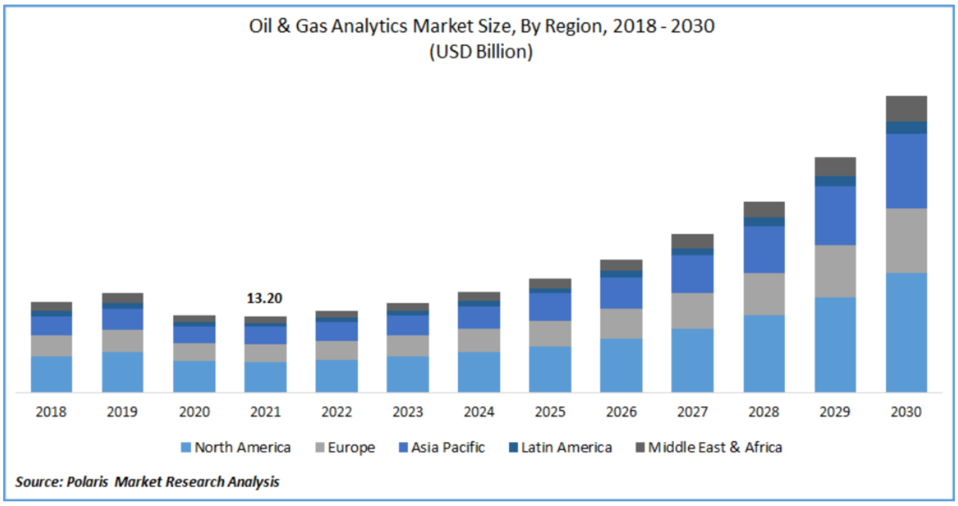
2. LITERATURE REVIEW

This chapter will be divided into five segments. The first part will explain the entire process for offshore platforms, using produced water, and technologies currently used for lithium and boron removal. Next, it is necessary to explain how 5 bottlenecks were selected for further analysis in this work. For that, two segments of the literature will be used: systematic review and mind maps. They were also essential for establishing and selecting data for analysis. Finally, Monte Carlo simulations, distribution approach through triangular distributions and confidence intervals were used for statistical analysis and proper data usage.

2.1. Produced water and DLBR process

Petroleum plays a crucial role as both an energy source and a revenue generator for numerous countries in the present era. It has been regarded as one of the most vital industrial activities of the twenty-first century (IGUNNU & CHEN, 2014). In Graph 1, it is possible to visualize the prospects of petroleum for the following decade, displaying an increase in the market size worldwide.

Graph 1. Market size for oil and gas worldwide.



Source: (POLARIS, 2022).

Despite its immense importance, petroleum production generates substantial waste, with wastewater comprising over 80% of the liquid waste (AZETSU-SCOTT et al., 2007). Presently, oil and gas companies treat produced water in different ways, such as mitigating the production of water through polymer gel or downhole water separators, discharging into the environment (following the legal procedures onshore and offshore), injection into formations, irrigation or reusing it in the petroleum industry after a minimum treatment (IGUNNU & CHEN, 2014). However, produced water is constituted of salts, oil, grease, BTEX (benzene, toluene, ethylbenzene, and xylenes), organic acids, inorganic and organic compounds, along with other chemicals (JIMÉNEZ et al., 2018); that can be explored financially when extracted from produced water. For this project, the analysis will focus on extracting two chemicals, lithium and boron.

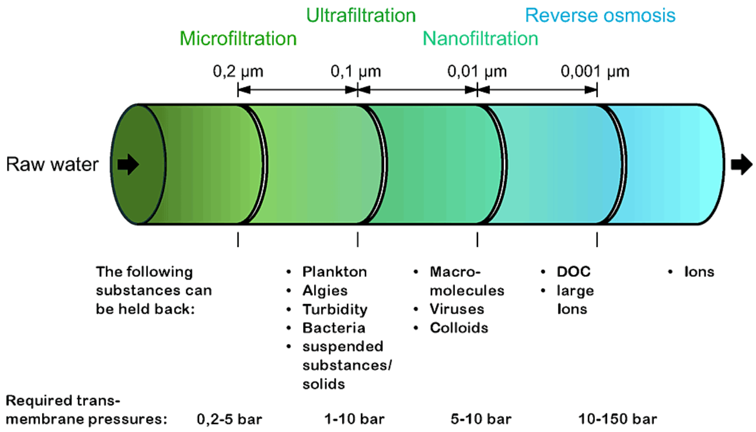
There are different technologies available for the treatment of produced water. For example, there are thermal technologies available for distillation (U.S. BUREAU OF RECLAMATION, 2003), hydrocyclones for the removal of particles (SVAROVSKY, 1992) and media filtration treatment for the removal of oil and grease (COLORADO SCHOOL OF MINES, 2009). This project will focus on two leading technologies for DLBR: electrochemical and membrane.

Since produced water has relatively good conductivity, it can be considered a potential electrolyte. Due to that, electrochemistry can be considered the future for produced water treatment, allowing for clean water production and recovery of valuable ions with minimal or inexistent impact on the environment (IGUNNU & CHEN, 2014). Membrane treatment will also be analyzed due to being one of the finest techniques available currently for produced water. However, this technology has the disadvantage of being vulnerable to membrane fouling, which can intercept or completely block pores of the membrane while decreasing the technology`s lifetime (FOULADITAJAR et al., 2013).

2.1.1. Membranes

The membrane process for filtration is a technique that utilizes a semi-permeable membrane to separate particles and contaminants from a fluid stream. The membrane acts as a barrier, allowing certain substances to pass through while retaining others based on size, shape, and charge. In membrane filtration, the fluid to be treated is passed through the membrane, and the contaminants or particles larger than the membrane's pore size are retained and separated from the purified fluid. Due to that, there are 4 main styles for membrane filtering: microfiltration (MF), ultrafiltration (UF), reverse osmosis (RO) and nanofiltration (NF) (IGUNNU & CHEN, 2014). However, MF and UF are primarily used for larger particles, such as macromolecules and suspended solids, while RO and NF are widely used for ions and organic molecules. Therefore, only the last two are usually studied for extraction of boron. Figure 1 displays a detailed analysis of each type of membrane treatment.

Figure 1. Types of membrane filtration.



Source: (HYDROGROUP, 2023).

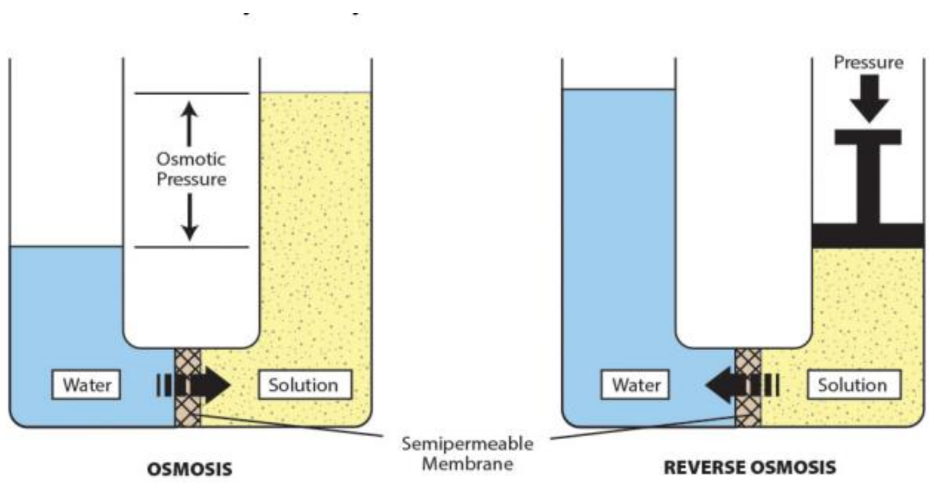
2.1.1.1. Nanofiltration

During nanofiltration, the fluid is passed through the nanofiltration membrane under pressure. The membrane acts as a selective barrier, allowing specific components to pass while retaining larger particles and ions. Nanofiltration membranes have a pore size range of approximately 0.001 to 0.01 micrometers, allowing them to effectively remove particles such as bacteria, viruses, organic molecules, and ions. A more critical definition can be formulated by the following parameters: pore diameter less than 2nm, substantially higher rejection of divalent ions than monovalent ions and rejection of neutrals and positive ions related mainly to size and shape (ZHANG et al., 2022).

2.1.1.2. Reverse Osmosis

Reverse osmosis (RO) technology is used in water purification to filter out small particles, such as ionic compounds. Typical applications of RO technology are seawater, desalination, boiler feed water filtering, product rinsing, microelectronics production, laboratory testing, biotechnology, and other processes that require highly purified water (MCMORDIE STOUGHTON et al., 2013). Reverse osmosis, as its name suggests, is the reversal of osmosis. Osmosis is a natural process where water with a lower concentration of dissolved solids moves through a membrane towards an area with a higher concentration, driven by osmotic pressure, ultimately equalizing the solute concentration on both sides of the membrane. In the case of reverse osmosis technology, pressure is applied to a water stream to counteract the natural osmotic pressure. This pressurized feed water is pushed through a semi-permeable membrane, resulting in purified water on one side and a concentrated solution of dissolved solids. Figure 2 shows a schematic of how the processes can be differentiated.

Figure 2. Osmosis and reverse osmosis.



Source: (MCMORDIE STOUGHTON et al., 2013)

2.1.2. Electrochemical

An electrochemical process refers to a chemical reaction involving the transfer of electrons between substances, typically facilitated by an electrical current. It involves converting electrical energy into chemical energy or using chemical reactions to generate electricity. Electrochemical processes are based on the principles of oxidation and reduction, where one substance loses electrons (oxidation) and another gains electrons (reduction). The science of electrochemistry deals with the chemical changes in materials accompanying the passage of an electric current, or the reverse process in which an electric current is generated by a chemical reaction, as in a battery (KURZWEIL, 2009).

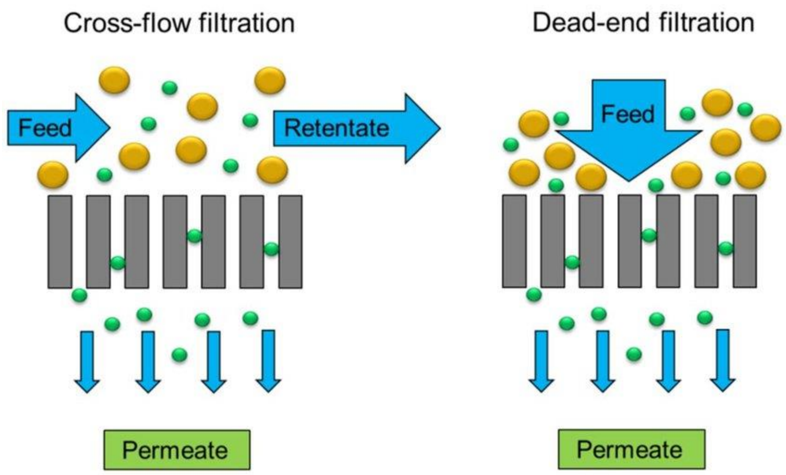
Produced water treatment typically does not use electrochemical methods, despite their extensive application in treating other types of wastewater. The established electrochemical treatment technologies for produced water are electrodialysis (ED) and electrodialysis reversal (EDR). These techniques effectively remove salts from produced water for irrigation purposes. However, it is important to note that produced water contains salts and detrimental substances such as heavy metals, oil, and solid particles. These contaminants can be equally harmful to the soil, in addition to the salt content. (IGUNNU & CHEN, 2014).

2.1.3. Types of filtration processes

It is also important to notice that membrane treatment can imply one of the following operating conditions: cross-flow or dead-end filtration. During the cross-flow filtration process, a water sample that has been prefiltered moves alongside the filtration membrane in a parallel direction, as illustrated in the left side of Figure 3. The filtration is driven by hydrostatic pressure, which pushes solutes that have a molecular size smaller than the membrane cutoff through the filter. This solution that successfully passes through the membrane is called the permeate, while the remaining solution is called the retentate. The retentate consists of smaller solutes and larger colloidal particles. It is typically carried along the membrane surface and recycled into the feed reservoir (ZHOU, 2012).

On the other side, in dead-end filtration, the feed solution moves in a perpendicular direction to the surface of the membrane. Unlike cross-flow filtration, which involves tangential flow, dead-end filtration has no separate reject stream. Instead, it has only a feed stream and a permeate stream, as depicted on the right side of Figure 3. The permeate carries all the substances in the feed solution toward the membrane surface, leading to the adsorption and deposition of solutes on the membrane surface (SINGH, 2015).

Figure 3. Cross-flow and dead-end filtration.



Source: (KETOLA, 2016).

The main disadvantage of dead-end filtration is extensive membrane fouling and concentration polarization, while cross-flow filtration is much less sensitive to fouling due to the sweeping effect of the tangential flowing of the fluid phase (NAGY, 2019).

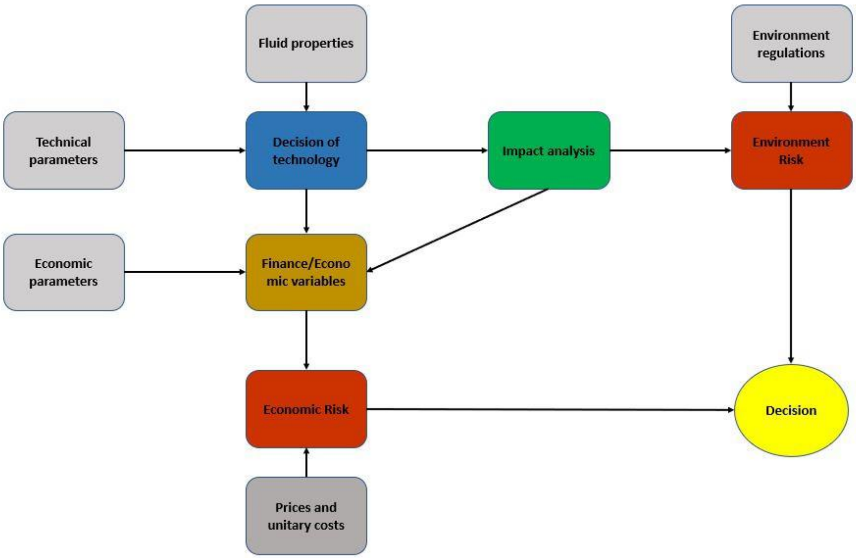
2.2. Mind Maps

Mind mapping is a technique that utilizes visual representations to capture and organize ideas. It involves creating a hierarchical structure, with a central topic at the core and related ideas branching out non-linearly. Each branch represents a key concept or subtopic; other branches can be added to capture more specific details (WETTE, 2017). This concept has shown several advantages. Mind maps stimulate creativity by encouraging a free-flowing, non-linear thought process. The visual nature of mind maps aids in making connections and identifying patterns between ideas. Secondly, mind maps improve information retention and recall as combining keywords, colors, and images enhances memory encoding and retrieval. Also, mind maps facilitate problem-solving by providing a clear overview of complex topics and enabling the identification of relationships and dependencies (GUERRERO, 2023). Due to these advantages, this methodology was used, combined with researchers from Escola Politecnica – University of Sao Paulo and oil field experts, to obtain a schematic of the principal variables influencing DLBR in offshore platforms for produced water.

By brainstorming the variables for the environmental and financial risk for DLBR in offshore platforms, pondering the weights between the connections of variables and final filtering of the significant variables for the problem, it was possible to obtain an exemplary schematic using mind maps in this first process.

In conclusion, it was possible to analyze the different factors that can influence the decision analysis of which technology would be more suitable for safely and financially positively removing these chemicals. In collaboration with a study proposed by researchers from Escola Politecnica- University of Sao Paulo and interviews with experts working in oil fields, the variables were organized and connected. The conclusion can be summarized with a diagram in Figure 4.

Figure 4. Influence of different variables for technology decision.



Source: Elaborated by the author.

The logic behind the diagram is explained as follows: for a technology decision, it is necessary to analyze the fluid characteristics of PW (such as temperature, salinity, and chemical distribution) and the technical parameters defined (such as type of material used, and energy source for extraction). As a result, different kinds of technology decisions will be selected and analyzed according to their impact on the environmental and financial economic-variables (variable and fixed costs for installation of technology). These financial-economic variables are compared with the prices of the chemicals extracted from PW to define the economic risk. The environmental and economic risks are analyzed to decide on technologies to implement in the offshore lithium and boron extraction platform.

Through Figure 4, experts and researchers could weigh the connection between variables and were able to define 5 bottlenecks, each with a different bottleneck that can influence environmental or economic risks. In this graduation work, four will be related to economic aspects, and one will be concerned with environmental regulations. These 5 bottlenecks are the primary analysis made in this project to evaluate and verify the possibility of implementing the technology through the respective bottleneck. However, to make this analysis, it was necessary to follow a methodology that could produce quality results while selecting information concisely and strongly connected with the selection of bottlenecks. For that, a systematic review was implemented, resulting in this study's initial point for analysis.

2.3. Systematic review

A systematic review aims to comprehensively identify, evaluate, and synthesize relevant research studies on a specific topic or research question. They provide a complete and unbiased summary of existing evidence, helping to inform policy decisions, clinical guidelines, and future research directions. Systematic reviews play a vital role in evidence-based practice by minimizing bias, maximizing transparency, and promoting high-quality evidence (LAVIS et al., 2004).

This section outlines the key steps in conducting a systematic review of the 5 bottlenecks selected. It includes developing inclusion and exclusion criteria, searching and selecting relevant studies, extracting, and analyzing data, assessing the quality of included studies, synthesizing findings, and interpreting the results. Each step is essential for ensuring the rigor and reliability of the review process (ROBINSON et al., 2014).

Systematic reviews offer several advantages over traditional literature reviews. They provide a comprehensive and transparent summary of the available evidence, minimize biases through strict methodology, and enhance generalizability by synthesizing data from multiple studies. Systematic reviews also help identify research gaps, highlight inconsistencies or controversies in the literature, and guide future research directions. While systematic reviews are a valuable research tool, they have limitations. Challenges may arise from the availability and quality of primary studies, study design and outcome heterogeneity, publication bias, and the substantial time and resources required to conduct a thorough review (BERO & JADAD, 1997). Acknowledging and addressing these limitations is crucial to ensure the validity and reliability of systematic review findings.

Although this project focuses on analyzing the 5 bottlenecks through data collection and critical analysis, a systematic review was primarily implemented to obtain the 5 bottlenecks, which will be verified in this study.

2.4. Monte Carlo Simulation

Monte Carlo Simulation is a computational technique that models and analyzes complex systems by generating many random samples or scenarios. For example, it is widely employed in finance, engineering, physics, and risk assessment (AVLIJAS, 2018).

The core idea behind Monte Carlo Simulation is to simulate a system or process multiple times, each time using randomly generated inputs or variables within their defined ranges. These inputs can represent uncertain factors, such as market prices, interest rates, or physical properties, that influence the behavior and outcomes of the studied system. By running numerous simulations, the technique provides a statistical representation of the possible outcomes, probabilities, and associated uncertainties (KROESE et al., 2014).

The Monte Carlo Simulation process involves the following key steps:

a) Define the problem: Clearly articulate the problem or system to be analyzed. Identify the input variables, their ranges, and the desired output or outcomes of interest.
b) Set up the model: Develop a mathematical or computational model representing the system and its relationships. This model should include the necessary equations, algorithms, and logical rules to simulate the system's behavior.
c) Generate random inputs: Randomly generate values for the input variables according to their defined distributions or probability functions. These random samples should cover the entire range of each variable and reflect their respective probability distributions.
d) Perform simulations: Run the model multiple times using the generated random inputs. Each simulation represents a potential realization of the system under the given input conditions. Capture the output or outcomes of interest for each simulation.
e) Analyze results: Analyze the collected simulation results to derive meaningful insights. This includes statistical analysis, such as calculating means, standard deviations, percentiles, and confidence intervals, to quantify the central tendency, variability, and uncertainty associated with the outcomes.

f) Interpret and communicate findings: Interpret the results in the context of the studied problem. Draw conclusions, make predictions, and communicate the findings effectively to stakeholders or decision-makers.

Monte Carlo Simulation offers several advantages. Firstly, it allows for considering uncertainty and variability in the analysis, providing a more realistic and comprehensive understanding of the system. Secondly, it can handle complex systems with nonlinear relationships and interdependencies. Thirdly, generating many simulations runs captures a wide range of potential outcomes, allowing for probabilistic assessments and risk evaluations (AVLIJAS, 2018), which can be proper for the environment and financial risk studied in this project.

However, Monte Carlo Simulation also has limitations. The accuracy of the results heavily relies on the quality and appropriateness of the input distributions and assumptions made in the model. It can be computationally intensive, requiring significant computational resources and time for complex systems with many variables and simulations. To increase the accuracy of this study, a considerable amount of research review was essential, reducing one of the disadvantages of this type of model. As for the second problem, this study will be using the software MATLAB®. The reason for using this software is that MATLAB® is an appropriate choice for statistical programs and analysis due to its extensive statistical functionality, intuitive syntax, visualization capabilities, interoperability, computational performance, and supportive user community. These features make MATLAB a versatile and proper tool for conducting statistical research, data analysis, and modeling in this work (MATLAB, 2023).

Therefore, Monte Carlo Simulation is a versatile and widely used technique for analyzing complex systems under uncertainty. By generating random samples and repeatedly simulating the system, it provides valuable insights into the range of possible outcomes, their probabilities, and associated risks. The technique has proven valuable for various disciplines' decision-making, risk management, and scenario analysis (IBM, 2023).

2.5. Defining distributions - Triangular distribution approach

To define distributions or probabilities for random inputs in Monte Carlo Simulation, obtaining a considerable amount of data for a single variable is fundamental, consequently defining the probability and distribution of the variable. However, most of the data obtained for this study reached a maximum, minimum and average value, with no sample size being available. Thus, triangular distribution will be used to define the probabilities of most of the variables in this study, attending step “c” of the Monte Carlo simulation.

The triangular distribution is a probability distribution often used in modeling uncertain variables where the values are constrained within a specified range. It is named as such because of its triangular shape, which is determined by three parameters: the minimum value (a), maximum value (c), and mode or peak (b) value (FAIRCHIELD et al., 2016). The mode represents the most likely or preferred outcome within the range.

In a triangular distribution, the probability density function (PDF) is highest at the mode and gradually decreases towards the minimum and maximum values. The distribution is symmetric if the mode is at the midpoint between the minimum and maximum values. However, it can be skewed if the mode is not at the midpoint (FORBES et al., 2011). Mathematically, to be considered a triangular distribution, it should follow these principles (Equations 1-3):

$$a \leq b \leq c \tag{1}$$

$$mean = \frac{a+b+c}{3} \tag{2}$$

$$var = \frac{a^2+b^2+c^2-ab-ac-bc}{18} \tag{3}$$

For Equation 2, "$mean$" is the average value and in Equation 3, "var" is the variance for the distribution. Also, the PDF function can be expressed as (Equation 4):

$$f(x|a,b,c) = \begin{cases} \dfrac{2(x-a)}{(c-a)(b-a)}; a \leq x \leq b \\ \\ \dfrac{2(c-x)}{(c-a)(c-b)}; b < x \leq c \\ \\ 0\ ;\ x < a\ , x > c \end{cases} \tag{4}$$

In conclusion, the triangular distribution is when there is a known relationship between the variable data but when there is relatively little data available to conduct a complete statistical analysis. Furthermore, the choice of parameters is crucial in accurately representing the uncertainty of a variable using a triangular distribution. It is often used in simulations when little is known about the data-generating process and is often referred to as a "lack of knowledge" distribution. The triangular distribution is ideal when the only data on hand are the maximum and minimum values and the most likely or preferred outcome. It is often used in business decision analysis (KISSELL & POSERINA, 2017).

2.6. Confidence Interval

To perform steps "e" and "f" of Monte Carlo Simulations, confidence intervals will be used as an alternative for analyzing the generated output of the simulations.

The confidence interval is a statistical concept that provides a range of plausible values for a population parameter based on sample data and a confidence level. It quantifies the uncertainty associated with estimating population parameters and is a fundamental tool in statistical inference. The concept of confidence interval stems from the fact that a point estimate, such as a sample mean or proportion, is unlikely to be precisely equal to the proper population parameter. The confidence interval provides a range of values within which the proper population parameter will likely lie. It is expressed as an interval with two endpoints: the lower and upper bound (ARFKEN et al., 2013).

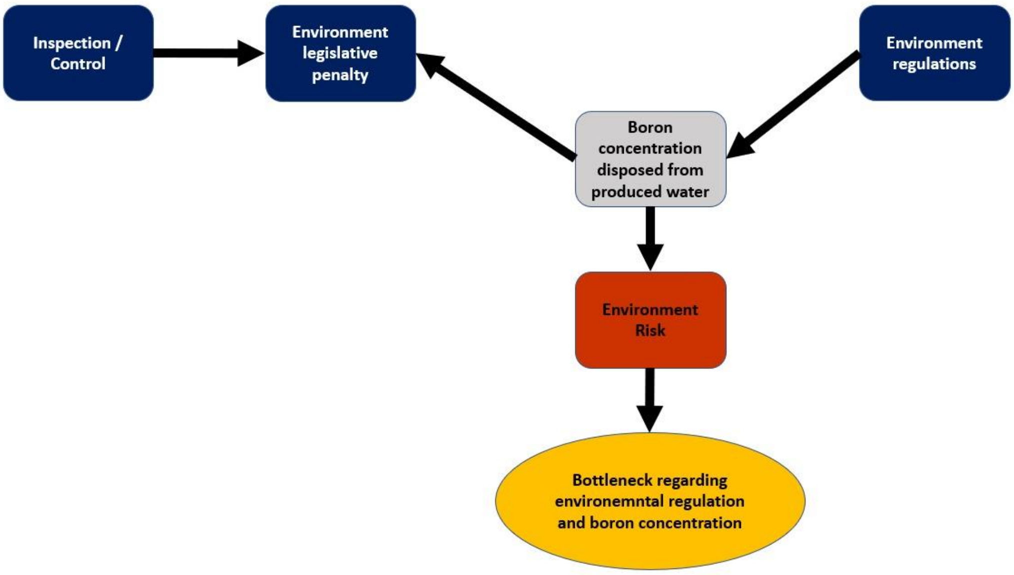
The confidence interval calculation depends on several factors, including the sample size, the variability of the data, and the chosen confidence level. The width of the interval is also influenced by it, for example. Increasing the sample size generally leads to narrower intervals as it reduces the uncertainty in the estimate. Similarly, reducing the variability of the data also results in narrower intervals. However, increasing the desired confidence level widens the interval to accommodate a higher level of certainty (BURRUSS & BRAY, 2005).

Therefore, the mechanism allows researchers to make inferences about the population based on limited sample data. By considering the range of plausible values, studies can assess the precision and reliability of their estimates. Additionally, an interval of confidence aids in hypothesis testing by determining whether the hypothesized value falls within the interval (ARFKEN et al., 2013). Thus, it is a proper alternative for handling the analysis of Monte Carlo simulations.

3. FIRST BOTTLENECK – ENVIRONMENTAL REGULATIONS

The first problem relates to environmental regulations and boron removal. The following Figure 5 displays a schematic for this bottleneck.

Figure 5. First bottleneck, environmental limitations.

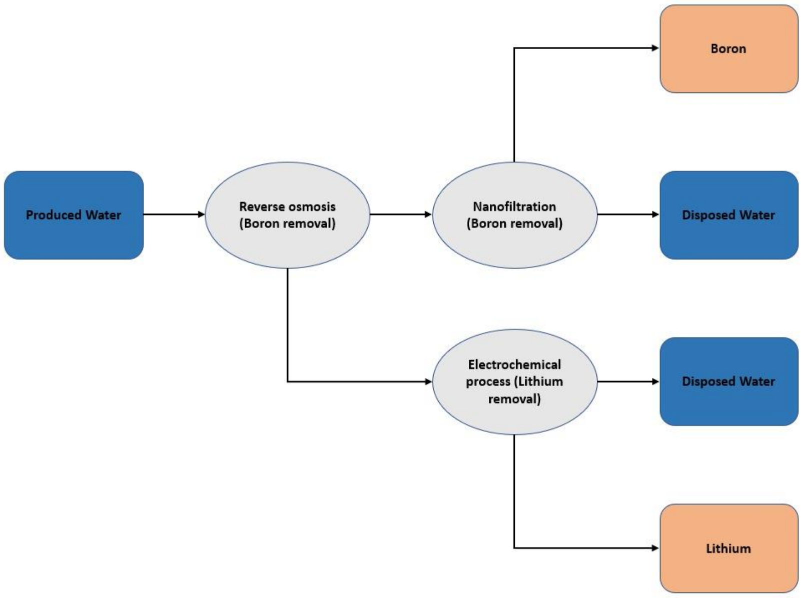


Source: Elaborated by the author.

It is possible to analyze from this bottleneck that it depends on how much boron from produced water can be dispersed in the environment to avoid penalties and fines while not resulting in degradation of the ambient and organisms present. For that, it is necessary to research environmental laws from different countries to avail the prospects of implementing the technology in different areas worldwide. It is also essential to analyze how the membrane filtration system can be improved to increase boron removal efficiency to accomplish the necessary environmental procedures.

Therefore, to analyze boron removal while considering membrane characteristics, this study will use a defined system to remove lithium and boron (Figure 6) directly. This schematic was proposed by researchers from *Escola Politécnica*- University of Sao Paulo and experts from the oil and gas field. To properly extract those chemicals, the produced water will first pass through reverse osmosis filtration for boron removal. After that treatment, the disposed water will be divided into two segments, and one will be focused on removing boron through membrane nanofiltration, while the other will be important for extracting lithium through the electrochemical process.

Figure 6. System for direct removal of lithium and boron.



Source: Elaborated by the author.

3.1. Data and analysis for boron concentration in disposed water

Since this first bottleneck only relates to how much boron can be rejected from this process to fulfill environmental regulations and avoid penalties or negatively impact the ecosystem, this analysis will focus on boron's membrane filtration (reverse osmosis and nanofiltration). Thus, it is necessary to collect data regarding boron concentration inside produced water, the percentage of boron rejection from reverse osmosis treatment and the rate of boron treatment from nanofiltration. Based on published reports about the topic, these variables can be specified in Table 1.

Table 1. Values for variables considered in boron treatment.

Variables	**Minimum value**	**Maximum value**	**Font**
Boron concentration in produced water	10 mg/L	30 mg/L	(EZECHI et al., 2014)
Boron treatment from reverse osmosis	$1 - 56.8\% = 43.2\%$	$1 - 40.7\% = 59.3\%$	(JARMA et al., 2021)
Boron treatment, from nanofiltration	$1 - 54.2\% = 45.8\%$	$1 - 44.1\% = 55.9\%$	(JARMA et al., 2021)

Source: Elaborated by the author.

The equation that describes the concentration of boron rejected from the treatment considers the variables presented in Table 2:

Table 2. Definiton of variables.

Description	**Variable**
The concentration of Boron treated in disposed water (final)	B_f

The concentration of boron in produced water (initial)	B_i
Percentage of treatment from reverse osmosis	P_{ro}
Percentage of treatment from nanofiltration	P_{nf}

Source: Elaborated by the author

And the equation can be synthesized as follows (Equation 5):

$$B_i \times P_{ro} \times P_{nf} = B_f \quad (5)$$

Due to the uncertainty of the input variables, and the importance of a model that can reproduce the results with precision and a broader perspective, Monte Carlo simulation will be used for analyzing the final concentration of the boron treated.

Therefore, with the equation above and considering the values for the variables obtained in Table 1 as triangular distributions, it is possible to create a sample in MATLAB® with size 10,000 by acknowledging random values between the minimum, average and maximum of each parameter, according to the probabilities within their triangular distributions, respectively.

Figure 7 shows how the software computed parameters and their values: lower, average (avg) and upper. The function “rng(‘default’)” allows for eliminating the random variables obtained through the triangular distribution for reproducibility.

Figure 7. Defining parameters in MATLAB®.

```
%Parameters
rng('default');  % For reproducibility

%Concentration produced water
lower1 = 10;
avg1 = 20;
upper1 = 30;

%Percentage reverse osmosis
lower2 =1-(0.568);
avg2 = 1-(0.4875);
upper2=1-(0.407);

%Percentage nanofiltration
lower3=1-(0.542);
avg3=1-(0.4915);
upper3=1-(0.441);
```

Source: Elaborated by the author.

After that, the sample size is created in Figure 8, and it is possible to create a triangular probability distribution for each parameter using their respective values.

Figure 8. Creating sample size and probability distribution in MATLAB®.

```
i=1;
f=10000;

pd1 = makedist('Triangular','A',lower1,'B',avg1,'C',upper1);
pd2 = makedist('Triangular','A',lower2,'B',avg2,'C',upper2);
pd3 = makedist('Triangular','A',lower3,'B',avg3,'C',upper3);
```

Source: Elaborated by the author.

Finally, in Figure 9, the program generates random numbers (r1, r2, r3) while considering their probability distribution and intervals and are multiplied with each other, following the equation model for this Monte Carlo simulation. The result of this individual part of the sample is stored inside the variable "result," and this iteration is made through a loop 10,000 times. The results are exposed in a histogram with the "histogram()" function.

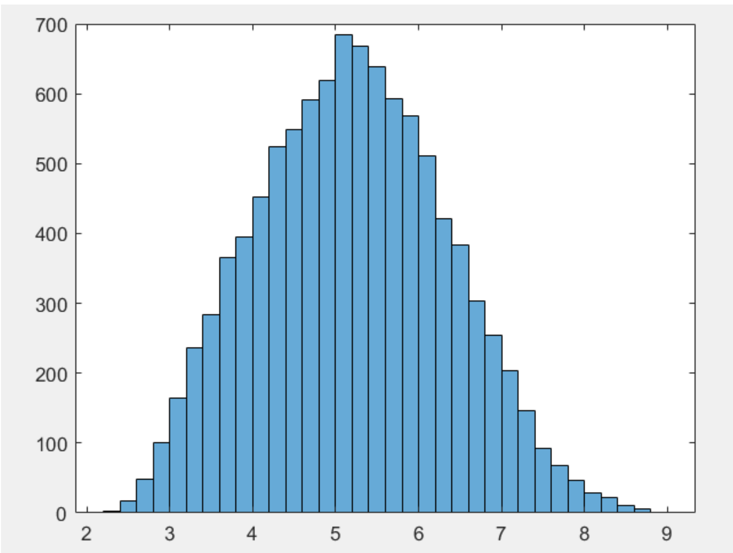
Figure 9. Generation of random numbers and histogram in MATLAB®, first bottleneck.

```
for test =i:f


    r1 = random(pd1,1,1)


    r2 = random(pd2,1,1)


    r3= random (pd3,1,1)

    rf= r1*r2*r3;
    result(test)=rf;
end

result = transpose (result)
histogram (result)
```

Source: Elaborated by the author.

The individual results of each sample are shown in Appendix A, with accumulative frequency, and can be expressed by the histogram in Graph 2.

Graph 2. Histogram analyzing boron concentration from produced water (mg/L) and frequency.



Source: Elaborated by the author.

The horizontal axis of the histogram represents the final boron concentration treated from produced water (mg/L). The vertical axis is the frequency of each value inside the sample of the Monte Carlo Simulation. Noticeably, the peak is obtained in the bin edges of 5.0 – 5.2, occurring 685 times, and the interval for the histogram fluctuates between 2.23 and 8.89, approximately. The histogram resembles a normal or student's t distribution, but since the sample has a size of 10,000, student's t should not be an option because it is mainly used for samples with small sizes. Adopting a normal distribution and with the function "fittest," it is possible to analyze details in Figure 10 to fit a distribution for the histogram.

Figure 10. Results for a normal distribution approximation of the histogram, first bottleneck.

```
>> curve = fitdist(result,'Normal')

curve =

  NormalDistribution

  Normal distribution
       mu = 5.20187   [5.17948, 5.22425]
    sigma = 1.14207
```

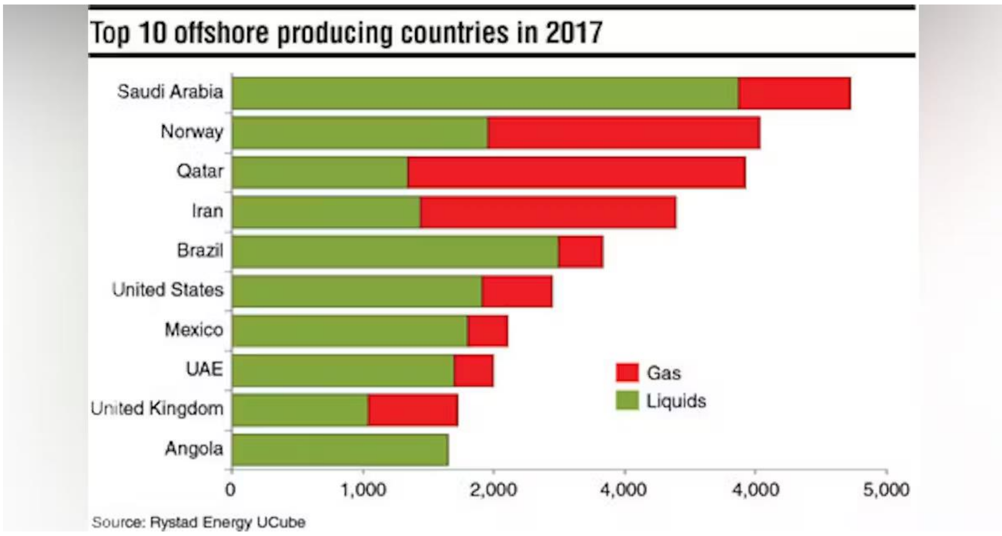
Source: Elaborated by the author.

The mean of the distribution is 5.202 mg/L, and the standard deviation is approximately 1.142. The function also can analyze the 95% confidence intervals for the distribution parameters, demonstrated between 5.17948 and 5.22425.

3.2. Data and analysis with environmental regulations

With the histogram and distribution analyzed, it is necessary to understand how different countries expect boron to be treated when considering environmental regulations. Since this study is related to produced water from offshore platforms, analyzing countries with the highest share, considering production is relevant. Graph 3 describes the 10 countries with the highest production on offshore platforms by MMb/d (Million barrels/day).

Graph 3. Top 10 countries for offshore production, country vs. oil and gas production (MMb/d).



Source: (RYSTAD ENERGY, 2017)

Thus, this study will focus on 3 of the biggest offshore-producing countries: Brazil, Saudi Arabia and Norway.

Brazil holds the 15th largest oil reserve worldwide and the 2nd in Latin America, with 12.2 billion barrels in offshore fields and 0.6 billion in onshore fields. The Campos and Santos oceanic sedimentary basins hold 94% of all Brazilian oil and hold the potential to be the largest offshore oil exploration fields in the following decades, mainly due to the oil located in the pre-salt layer (MARONEZE et al., 2014).

Saudi Arabia is the world’s largest offshore producer, with 13% of the global total has several sizeable offshore oil fields, including the Safaniya oil field, which produces between 1.1 and 1.5 MMb/D and is the highest-producing offshore field in the world (JOURNAL OF PETROLEUM TECHNOLOGY, 2016).

Finally, Norway is the world's 7th largest natural gas producer, supplying 3 percent of global gas consumption and ranks as the world´s 3rd largest exporter of natural gas. It is also a significant oil producer, accounting for 2.3 percent of global oil production in 2020. Ranking as the 10th largest oil exporter, the petroleum sector remains a crucial driver for the Norwegian economy. Also, this study can be very intriguing for this country since oil and gas production in Norway is only offshore (TRADE, 2022).

Thus, it will be verified how these countries' environmental regulations accommodate in the histogram (Graph 2). Searching from articles and scientific reports, it was noticeable that boron has different limitations in aqueous forms, depending on how the liquid will be used. For example, it can be used for irrigation and as curative water within certain limitations (CHRUSZCZ-LIPSKA et al., 2020). Therefore, to standardize regulations from different countries, the limitations will be according to the concentration of boron for disposal in the ocean, exposed in Table 3.

Table 3. Limitations of disposal of water in the ocean, for different countries.

Country	Limitation (mg/L)	Font
Brazil	5.0	(BRAZIL, 2005)
Saudi Arabia	5.4	(AL-RASHEED et al., 2017)
Norway	4.5	(ESCARABAJAL-HENAREJOS et al., 2021)

Source: Elaborated by the author.

Comparing Table 3 and the results in Figure 10, with a 95% confidence interval, it is noticeable that the system will only be functional in Saudi Arabia, which has limitations above the intervals 5.17948 and 5.22425.

3.3. Improving the system by increasing reverse osmosis efficiency

One method for improving the process and allowing the system to be adequate with the environmental regulations of other countries can be done by increasing the efficiency of the reverse osmosis boron removal treatment.

As explained previously, the boron extraction process mainly involves reverse osmosis and nanofiltration. Reverse osmosis commonly uses cellulose acetate (CA) or thin film composite (TFC) membranes. Table 4 explains the detailed characteristics of each membrane (MCMORDIE STOUGHTON et al., 2013).

Table 4. Differences between membranes for reverse osmosis.

Aspect	CA membrane	TFC membrane
Filtration of organic compounds	Low	High
Water flux	Medium	High
pH tolerance	4 - 8	2 - 11
Temperature stability	Max 35 °C	Max 45 °C
Oxidant tolerance	High	Low
Compaction Tendency	High	Low
Cost	Low	High

Source: Elaborated by the author.

Analyzing both membranes, it is interesting to notice that TFC membranes are highly recommended for boron removal since they have increased efficiency for the filtration of organic compounds and are adequate for higher water flux from produced water, with better resistance for variations in temperature and pH. This flexibility allows for implementing the technology in different geological locations, avoiding environmental legal procedures. However, they are costly when compared to CA membranes.

It is also essential to analyze how to improve the recovery rate for membrane filtration. For that, verifying the factors influencing its efficiency is crucial: feed water pressure, fouling and membrane degradation.

When other factors, such as osmotic pressure and water temperature, are kept constant, the water flux across the membrane increases directly with increased feed water pressure. Additionally, higher feed water pressure leads to increased rejection of dissolved solids. If the water pressure remains within the maximum designed pressure range of the system, higher pressure results in better quantity and quality of the produced water (MCMORDIE STOUGHTON et al., 2013).

Membrane fouling in most membrane processes typically occurs due to the precipitation and deposition of molecules or particles on the surface or pores of the membrane. This fouling has several consequences, including increased resistance to membrane separation, decreased productivity, and potential changes in membrane selectivity. These effects impact the separation of the desired components in the feed, resulting in unstable product quality and poor recovery. The fouling process usually involves pore blocking, the aggregation of solutes leading to forming of a cake or gel layer on the membrane surface, and adsorption, exacerbated by concentration polarization and convective forces acting on and through the membrane. There are three main groups of factors that contribute to membrane fouling. The first group includes membrane properties, such as the material used to construct the membrane. The second group encompasses the properties of the feed solution, including its composition, concentration, pH, and ionic conditions. The last factor is related to the flux direction (cross-flow or dead-end scenario). These factors can influence the likelihood and extent of fouling (LI & CHEN, 2010). Lastly, membrane degradation is related to the membrane's lifespan, which can be influenced by different variables, such as the necessary maintenance, flux direction selected, components inside the feed used for filtering, and external chemicals to facilitate the removal of chemicals.

Considering those activities for improving the rate of boron removal with the Monte Carlo simulations exercised in MATLAB®, it is possible to create scenarios to quantify how the increase in reverse osmosis treatment efficiency would attend environmental protocols from other countries besides Saudi Arabia: 5%, 10% and precise increment to attend legislations from Brazil and Norway.

Increasing the efficiency of the reverse osmosis process by 5% (Figure 11):

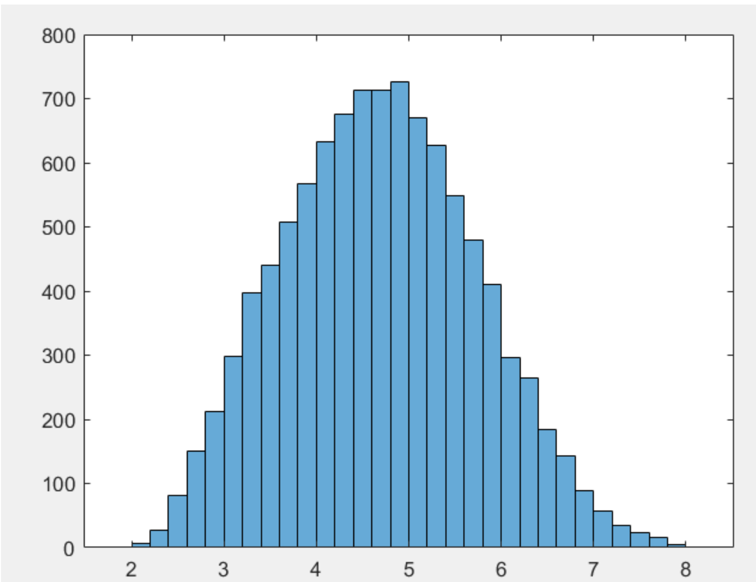
Figure 11. Improvement of reverse osmosis by 5%.

```
%Percentage reverse osmosis
lower2 =1-(0.568+0.05);
avg2 = 1-(0.4875+0.05);
upper2=1-(0.407+0.05);
```

Source: Elaborated by the author.

The results are displayed in the histogram of Graph 4, and the values with accumulated frequency in Appendix B:

Graph 4. Histogram for reverse osmosis improvement by 5%, frequency vs. boron concentration (mg/L).



Source: Elaborated by the author.

Finally, accounting for a normal distribution (Figure 12):

Figure 12. Output for normal distribution, with improvement in reverse osmosis by 5%.

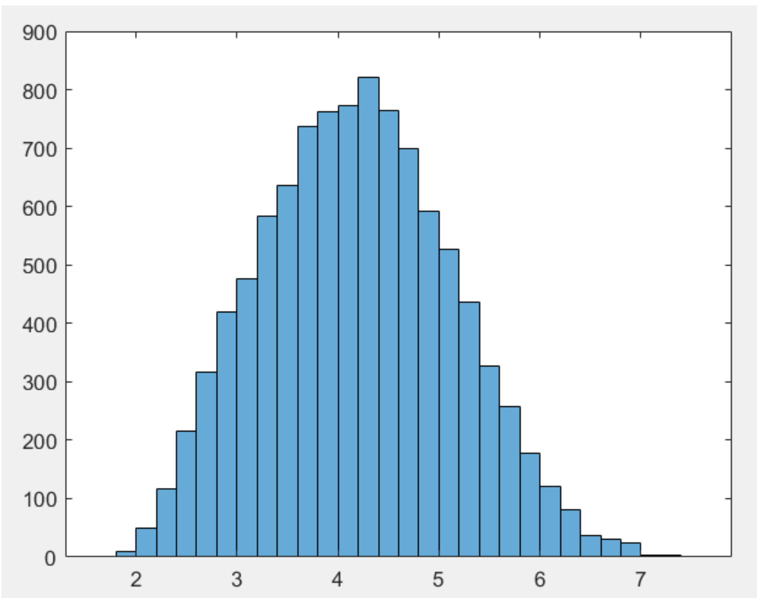
```
curve =

  NormalDistribution

  Normal distribution
       mu = 4.6946   [4.67418, 4.71503]
    sigma = 1.0419   [1.02766, 1.05654]
```

Source: Elaborated by the author.

Therefore, an increase of 5% in the reverse osmosis process reduces the 95% confidence interval for values of boron concentration between 4.67418 and 4.71503, which is below the limit of disposing of water of Brazil (5mg/L). However, it does not yet compromise with Norway's environmental regulations (4.5mg/L).

If an increase of 10% were implemented in the reverse osmosis system, the histogram would be (Graph 5). Values and accumulated frequency are in Appendix C.

Graph 5. Histogram for reverse osmosis improvement by 10%, frequency vs. boron concentration(mg/L).



Source: Elaborated by the author.

Approaching a normal distribution for the histogram (Figure 13):

Figure 13. Output for normal distribution, with improvement in reverse osmosis by 10%.

```
curve =

  NormalDistribution

  Normal distribution
       mu =  4.18734   [4.16886, 4.20582]
    sigma = 0.942897   [0.930009, 0.956151]
```

Source: Elaborated by the author.

Consequently, if a 10% increase enhances the reverse osmosis process, the boron concentration is estimated to fall within a narrower range with a 95% confidence interval between 4.16886 and 4.20582. These values are below the permissible limit for water disposal in the Norwegian Sea, which is 4.5 mg/L.

By adopting a constant increment value, initially 0, until the upper limit of the reverse osmosis treatment reaches 100%, and with the environmental restrictions of Brazil and Norway inputted in MATLAB®, it is possible to adopt a trial-and-error configuration, which resulted in the precise efficiency improvement necessary to satisfy the environmental requirements of each country.

For Brazil, the output for x to attend restrictions is approximately 2.3%, which can be detailed in Figure 14, and further data are in Appendix D.

Figure 14. Output for normal distribution approach, Brazil scenario.

```
curve =

  NormalDistribution

  Normal distribution
       mu = 4.96853   [4.94704, 4.99001]
    sigma = 1.09587   [1.08089, 1.11127]
```

Source: Elaborated by the author.

Lastly, Norway's efficiency in reverse osmosis should be increased by approximately 7.2% to respect environmental regulations of 4.5 mg/L of boron concentration in seawater. Figure 15 demonstrates the 95 % confidence interval of a lower limit of 4.45184 and an upper limit of 4.49097. Values and accumulated frequency to generate the normal distribution approach are displayed in Appendix E.

Figure 15. Output for normal distribution approach, Norway scenario.

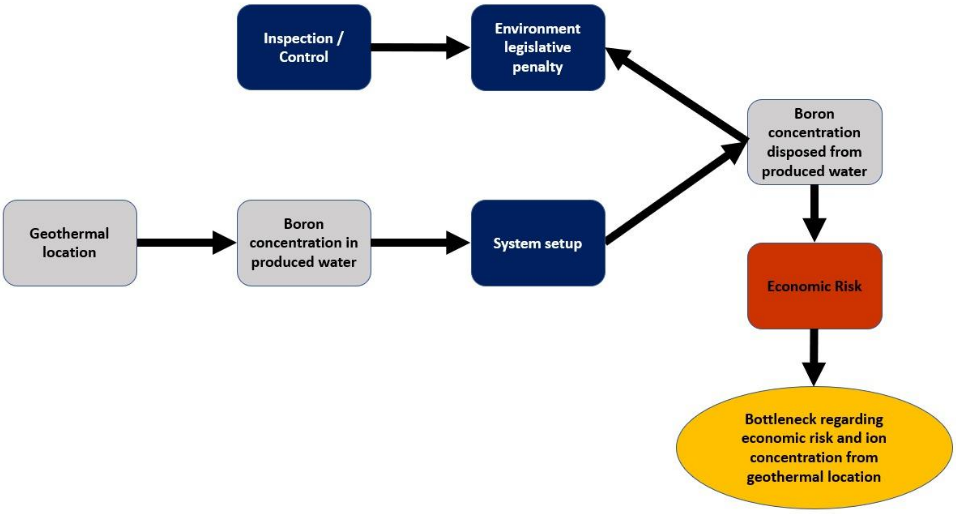
```
curve =

  NormalDistribution

  Normal distribution
       mu =  4.47141   [4.45184, 4.49097]
    sigma = 0.998173   [0.984529, 1.0122]
```

Source: Elaborated by the author.

4. SECOND BOTTLENECK – CONCENTRATION ISSUE

The second bottleneck connects boron concentration and economic risks by analyzing the risk of acquiring penalties for not following environmental regulations, as demonstrated in Figure 16.

Figure 16. Second bottleneck, concentration issue.



Source: Elaborated by the author.

The problem consists of location, especially how different geothermal extractions positions can result in various ion concentrations for produced water. Consequently, maintaining the efficiency for reverse osmosis and nanofiltration, the final concentration of boron in disposed water will be limited by the initial concentration of the chemical. This can result in penalties or taxes, which can compromise the economic feasibility of the system.

Although the variables for the Monte Carlo simulation are the same as the ones used for defining the first bottleneck, the equation is different since, in this analysis, the final concentration (B_f) is considered an input, and the initial concentration of boron (B_i) is the output desired to determine proper geothermal locations to extract oil and gas. Thus, the equation to create Monte Carlo simulations will be (Equation 6):

$$B_i = \frac{B_f}{P_{ro} \times P_{nf}} \tag{6}$$

The efficiencies of reverse osmosis and nanofiltration were already extracted (Table 1). For describing B_f as an input value, a triangular distribution will be considered using the data from Table 3. Therefore, the lower mode and upper bound will be 4.5 mg/L, 5.0 mg/L and 5.4 mg/L. Implementing this model in the code will result in Figure 17, which is similar to the one implemented for the first bottleneck, with significant differences in the input data (lines 4 – 7) and the mathematical equation for the simulation (line 34):

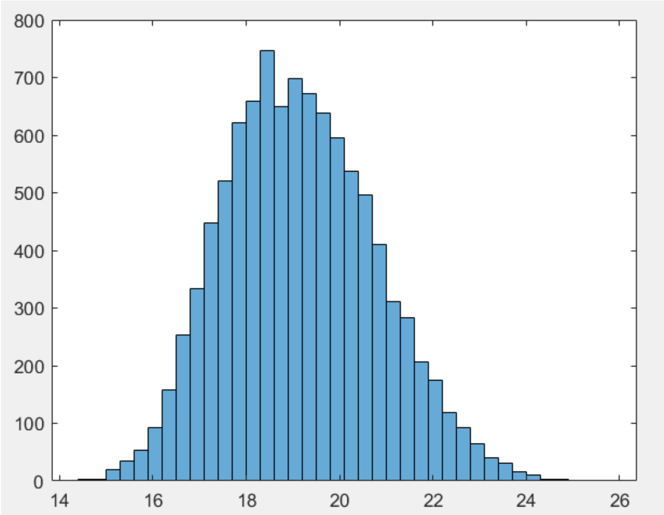
Figure 17. Code for second bottleneck.

```
4   %Final concentration produced water
5   lower1 = 4.5;
6   avg1 = 5;
7   upper1 = 5.4;
8
9   %Percentage reverse osmosis
10  lower2 =1-(0.568);
11  avg2 = 1-(0.4875);
12  upper2=1-(0.407);
13
14  %Percentage nanofiltration
15  lower3=1-(0.542);
16  avg3=1-(0.4915);
17  upper3=1-(0.441);
18
19  i=1;
20  f=10000;
21
22  pd1 = makedist('Triangular','A',lower1,'B',avg1,'C',upper1);
23  pd2 = makedist('Triangular','A',lower2,'B',avg2,'C',upper2);
24  pd3 = makedist('Triangular','A',lower3,'B',avg3,'C',upper3);
25
26  for test =i:f
27
28      r1 = random(pd1,1,1);
29
30      r2 = random(pd2,1,1);
31
32      r3= random (pd3,1,1);
33
34      rf= r1/(r2*r3);
35      result(test)=rf;
36  end
```

Source: Elaborated by the author.

Moreover, the resulting histogram can be displayed in Graph 6 and data details in Appendix F:

Graph 6. Histogram for second bottleneck, frequency vs. initial boron concentration (mg/L).



Source: Elaborated by the author.

Aggregating the data to fit a distribution for the 10,000 outputs, using the function "fitdist" in MATLAB®, will result in a mean and sigma of approximately 19.16 mg/L and 1.61 mg/L. This data makes it possible to calculate the 95% confidence interval, resulting in values between 19.13 mg/L and 19,20 mg/L. A conclusion for specifically this bottleneck, in general, geothermal positions that can generate produced water at a 19 mg/L level or below for concentration of boron are safe to install this DLBR system when considering avoiding penalties for environmental regulations.

Furthermore, this research can be more precise if the exact limits of disposal of boron in seawater for determined countries can be collected. Using as an example the already extracted data from Brazil, Saudi Arabia, and Norway, the variable B_f in the equation for the model will be considered a constant since each country has a precise regulation for discarding boron in seawater. The results are determined in Table 5:

Table 5. Proper boron concentration by country.

Country	Boron regulations for disposal (mg/L)	Mean of initial boron concentration necessary in PW (mg/L)	95% Confidence interval (mg/L)	
			Lower	Upper
Brazil	5,0	19,30	19,27	19,34
Saudi Arabia	5,4	20,85	20,82	20,88
Norway	4,5	17,37	17,34	17,40

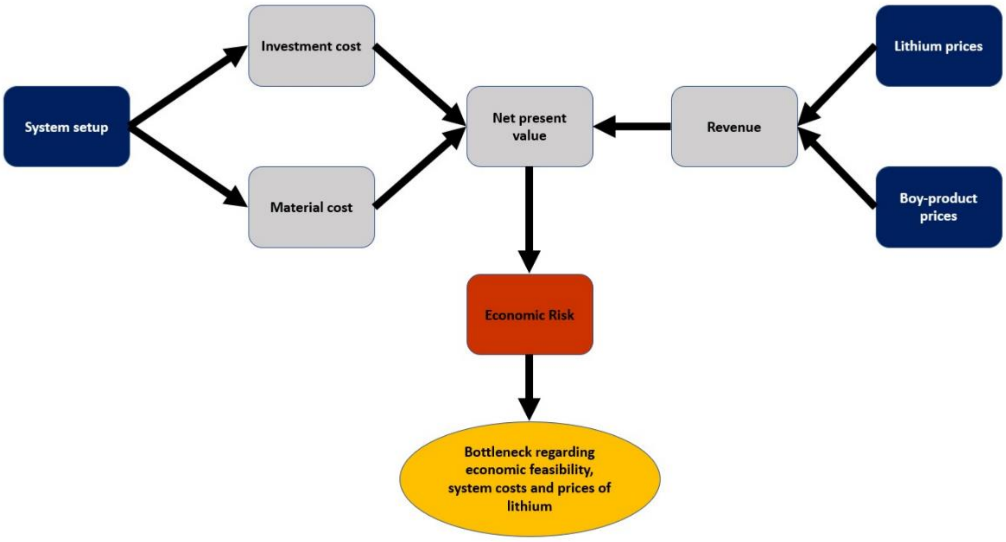
Source: Elaborated by the author

Therefore, acknowledging the environmental limitations of boron for each country combined with the analysis for this bottleneck can determine limits for geological extraction of oil and gas according to the concentration of boron present in produced water.

5. THIRD BOTTLENECK – MARKET ISSUE

The next problem relates to the economic and financial feasibility of directly removing lithium and boron from produced water, investigating revenue and costs. The following Figure 18 displays a schematic for this bottleneck.

Figure 18. Third bottleneck, market issue.



Source: Elaborated by the author.

The bottleneck demonstrates how the economic feasibility is determined by diverse variables, being a bottleneck in this issue. System expenses related to investment and materials are necessary to implement the DLBR for produced water. On the other hand, the revenue is obtained from lithium prices. To analyze this issue, it is necessary to search for lithium price fluctuations and cost estimations of technologies already implemented or researched for DLBR.

5.1. Lithium prices

To verify historical data, access through the Bloomberg Terminal allowed the analysis of lithium prices across years, resulting in Graph 7. The Bloomberg Terminal is designed to deliver rich data across market sectors and workflows through one unified system that connects you to the data, news, analytics, and people you need to make fast, effective decisions (BLOOMBERG, 2023).

The horizontal axis is expressed by the time between 2020 and 2023 (described in the left-upper corner of Graph 7). The vertical axis represents the price of lithium in US$/kg.

Graph 7. Lithium price fluctuations, lithium price (USD/ton) vs. time.

Source: (BLOOMBERG, 2023).

From Graph 7, data for the upper (85,000 USD/kg) and lower bound (13,190 US$/kg), along with an average value (54,100 USD/kg), are obtained.

5.2. System costs

To estimate system costs for this bottleneck, literature research was conducted to analyze different technologies available for lithium removal, along with their respective costs. The essential data for the simulation are summarized in Table 6.

Table 6. Summary of system costs for different technologies.

Source	Technology decision	Costs (USD/ton)	Reference
Geothermal brines	Direct extraction	3,217 - 4,178	(WARREN, 2021)
Diluted brines	Electrochemical ion exchange	6,000	(PALAGONIA et al., 2020)
Seawater	-	15,435 - 22,000	(YAKSIC & TILTON, 2009)

Source: Elaborated by the author

The first reference explains various methods and approaches suggested to extract lithium from geothermal directly, and other brines. These techniques can be classified into adsorption, ion exchange, and solvent extraction, allowing for comparison and evaluation. Geothermal brines, particularly those rich in lithium, hold immense potential as an abundant and untapped resource that could be developed to establish a reliable domestic supply. Direct lithium extraction technologies encompass diverse techniques that can be employed to extract lithium from brines. Implementing these techniques presents an opportunity to enhance sustainability and minimize environmental impacts compared to traditional methods, such as evaporative ponds and hard rock mining approaches used for lithium production. By exploring and adopting direct lithium removal technologies, the potential exists to modernize lithium extraction, making it more efficient, environmentally friendly, and economically viable (WARREN, 2021).

The next article starts by elaborating a critique of the current system of removing lithium for being slow, inefficient and with strong environmental impact. Therefore, it proposes a new technology called "electrochemical ion pumping," based on the selective electrochemical capture of lithium cations, to mitigate or reduce the disadvantages of the current system (PALAGONIA ET AL., 2020). To analyze that, the article verifies the ecosystem impacts and economic costs presented in Table 6.

Lastly, the final reference estimates lithium extraction costs from seawater by knowledge about the shape of the cumulative availability curve, which shows the amount of a mineral commodity that can be recovered profitably at different prices from different types of mineral deposits under current conditions (that is, current technology, prevailing labor and other input prices, and so on). Despite the inherent uncertainties surrounding the future growth in lithium demand as well as the uncertainties regarding the future cost-reducing effects of new production technologies, the shape of the lithium cumulative availability curve obtained in this article indicates that depletion is not likely to pose a serious problem over the rest of this century and well beyond (YAKSIC & TILTON, 2009).

5.3. Monte Carlo Simulation

Acknowledging the lowest and highest values for system costs encountered and considering the mode value as the arithmetic average of all the values presented in Table 6. These values can be combined with the historical data obtained for lithium prices in the following Table 7:

Table 7. Data for Monte Carlo simulation, third bottleneck.

Variable	**Lower bound (USD/ton)**	**Average (USD/ton)**	**Upper bound (USD/ton)**

Lithium prices	13 190	54 100	85 000
System costs	3 217	10 166	22 000

Source: Elaborated by the author.

These variables can be proposed as triangular distributions for defining the random input for the simulation. The next step is generating the mathematical equation that models the simulation. To do that, the hypothesis is to verify if it is economically feasible to use lithium from produced water as a financial asset, despite system costs being necessary to obtain this chemical. Therefore, the mathematical model can be defined by (Table 8):

Table 8. Variables for third bottleneck.

Description	**Type of distribution**	**Type of data**	**Name**
Lithium prices	Triangular distribution	Input	Li
System costs	Triangular distribution	Input	S
Concentration of lithium from produced water	Fixed value	Input	C_l

Volume of produced water	Fixed value	Input	V
Economic feasibility	-	Output	E

Source: Elaborated by the author.

Since this project analysis each bottleneck separately, the focus of this topic is to analyze how fluctuations in terms of the responsible agents for originating the bottleneck respond through simulations. Therefore, the concentration of lithium in produced water (C_l) and volume of produced water (V) will be considered fixed (Equation 7).

$$E = (V \times C_l \times Li) - (V \times C_l \times S) = V \times C_l(Li - S) \quad (7)$$

Since C_l and V are constant positive numbers; they can be reduced to the constant α (Equation 8):

$$E = \alpha(Li - S), \qquad \alpha = V \times C \quad (8)$$

However, whereas that α is also a constant non-null positive number, it will not influence the analysis of this bottleneck since what this study is trying to analyze is if it is economically feasible ($E \geq 0$) or not ($E < 0$). Finally, the Monte Carlo simulation can be reduced for the Equation 9:

$$E' = Li - S \quad (9)$$

With the data obtained and the mathematical model defined, developing a Monte Carlo simulation in MATLAB® is possible. Figure 19 displays the code for generating the 10,000 random outputs and fitting the distribution for the resulting histogram.

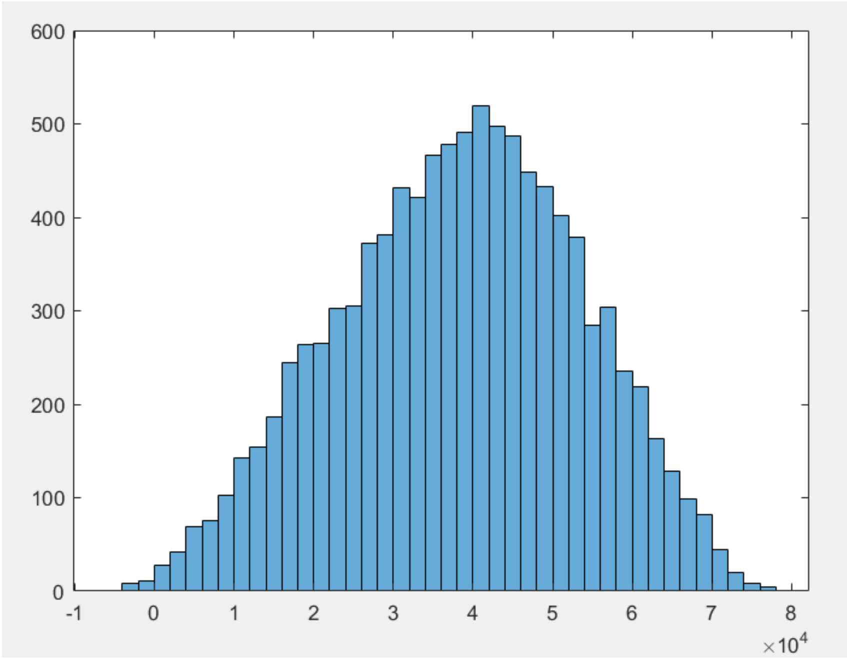
Figure 19. Code for third bottleneck.

```
%Parameters
rng('default');  % For reproducibility

%Lithium price
lower1 = 13190;
avg1 =54100;
upper1 = 85000;

%System costs
lower2 =3217;
avg2 = 11473.3;
upper2=22000;

i=1;
f=10000;

pd1 = makedist('Triangular','A',lower1,'B',avg1,'C',upper1);
pd2 = makedist('Triangular','A',lower2,'B',avg2,'C',upper2);

for test =i:f

    r1 = random(pd1,1,1);

    r2 = random(pd2,1,1);

    rf= r1-r2;
    result(test)=rf;
end

result1 = transpose (result);
histogram (result)
```

Source: Elaborated by the author.

Then, with the output data collected for E', the histogram is created, as in Graph 8. The horizontal axis is measured by USD/ton, and the vertical axis represents the output frequency. Data details are presented in Appendix G.

Graph 8. Histogram for the third bottleneck, frequency vs. profit (USD/ton).



Source: Elaborated by the author.

Using the function "fitdist" from MATLAB®, the software will fit a proper distribution and calculate the mean, sigma and 95% confidence interval. Figure 20 demonstrates the outputs of the program:

Figure 20. Output of the code, third bottleneck.

```
   mu = 38334.5    [38036.8, 38632.3]
sigma = 15190.2
```

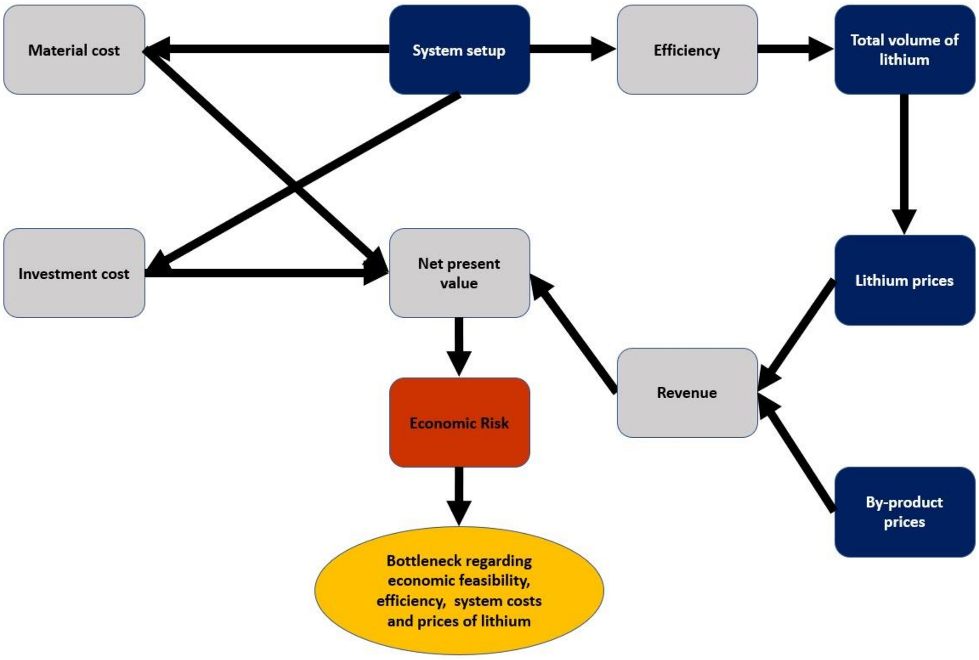
Source: Elaborated by the author.

Since the results are positive (95% confidence interval limited by 38 036,8 USD/ton and 38 632,3 USD/ton), the economic variable can be considered feasible since lithium prices surpass system costs. However, this bottleneck does not integrate with other variables that can reduce the revenue obtained by lithium prices, such as the system's efficiency, which will be described in the next bottleneck.

6. FOURTH BOTTLENECK – EFFICIENCY ISSUE

Related to the previous bottleneck, this bottleneck identifies the efficiency of the system setup for direct removing lithium from produced water as an obstacle that can induce economic risks, consequently not being financially feasible. Figure 21 displays in detail the bottleneck.

Figure 21. Fourth bottleneck, efficiency issue.



Source: Elaborated by the author.

6.1. Data collection

Therefore, this bottleneck reduces the revenue obtained in the previous bottleneck by reducing the volume of lithium that can be extracted from produced water. To analyze this case, data collection was made for available technologies and their respective efficiencies (Table 9).

Table 9. Summary of efficiency values, fourth bottleneck

Source	Efficiency (%)	Reference
Groundwater	38 - 95	(AHMAD et al., 2022)
Shale gas produced water	42.3 – 44.9	(LEE & CHUNG, 2020)
Aqueous solution	80%	(OYARCE et al., 2022)

Source: Elaborated by the author.

The first article studies the use of different adsorbents to remove lithium: activated carbon, bentonite, roasted date pits, and modified-roasted date. Along with these products, the data was tested for different pH, concentrations, and temperatures. Results demonstrated that the considerable interval difference between maximum and minimum efficiency for this technology test is mainly related to temperature and adsorbent utilized. This method's maximum efficiency is obtained using activated carbon as an adsorbent and a temperature of 35 °C (AHMAD ET AL., 2022).

The following article uses shale gas-produced water to recover lithium through solvent extraction technology. The influence of different types and concentrations of alkane in the solvent extraction process was studied to observe the efficiency. It was found that the chain length of alkanes affected lithium recovery efficiency. Also, the rise in hexane concentrations as a solvent resulted in incremented values for the method's efficiency (LEE & CHUNG, 2020).

The final research uses different polymers to identify which produces the most relevant results for lithium removal. To verify that, four different polymers, combined with solutions with different pH and lithium concentrations, were tested. The results demonstrated that lithium removal was 80% (OYARCE ET AL., 2022).

6.2. Monte Carlo Simulation

Similarly, as the third bottleneck, the data for efficiency will be considered as a triangular distribution, using the arithmetic average as mode and lower and upper bound, minimum and maximum values collected, respectively (Table 10).

Table 10. Values for efficiency variable.

Variable	Lower bound	Mode	Upper bound
Efficiency (e)	38%	60%	95%

Source: Elaborated by the author.

Adapting the mathematical equation from the third bottleneck for efficiency issue (Equation 10):

$$E' = (e \times Li) - S \tag{10}$$

Applying the equation and inputs in MATLAB® generates the following code (Figure 22):

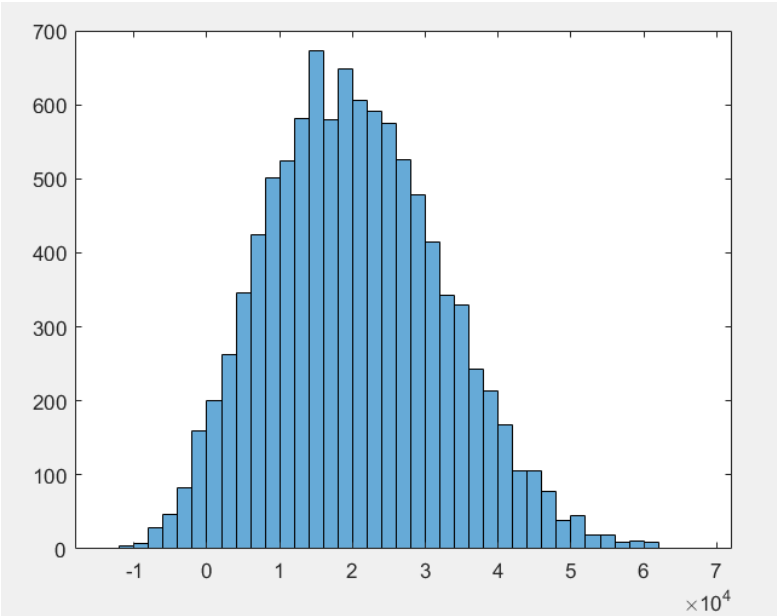
Figure 22. Code for fourth bottleneck.

```
%Efficiency
lower1 = 0.38;
avg1 = 0.6;
upper1 = 0.95;

%Lithium prices
lower2 =13190;
avg2 = 54100;
upper2= 85000;

%System costs
lower3=3217;
avg3=11473.3;
upper3=22000;

i=1;
f=10000;

pd1 = makedist('Triangular','A',lower1,'B',avg1,'C',upper1);
pd2 = makedist('Triangular','A',lower2,'B',avg2,'C',upper2);
pd3 = makedist('Triangular','A',lower3,'B',avg3,'C',upper3);

for test =i:f

    r1 = random(pd1,1,1);

    r2 = random(pd2,1,1);

    r3= random (pd3,1,1);

    rf= (r1*r2)-r3;
    result(test)=rf;
```

Source: Elaborated by the author.

Generating 10,000 data for the output sample, the histogram is represented in Graph 9. The horizontal axis is measured in USD/ton, and the vertical axis represents frequency. Data details are in Appendix H.

Graph 9. Histogram for fourth bottleneck, frequency vs. profit (USD/ton).



Source: Elaborated by the author.

The MATLAB® software also has the potential to fit a distribution in the histogram and produce the 95% confidence interval for the sample. By fitting a normal distribution, the results are displayed in Figure 23.

Figure 23. Output of the code, fourth bottleneck.

```
NormalDistribution

Normal distribution
     mu = 20439.7   [20203.4, 20675.9]
  sigma =    12053
```

Source: Elaborated by the author.

In conclusion, the results displayed a reduction of approximately 17.8 thousand USD/ton compared to the analysis in the third bottleneck. However, the output generated considering the efficiency issue still obtained positive results (intervals between 20203.4 USD/ton and 20675.9 USD/ton), demonstrating that the system is still financially viable.

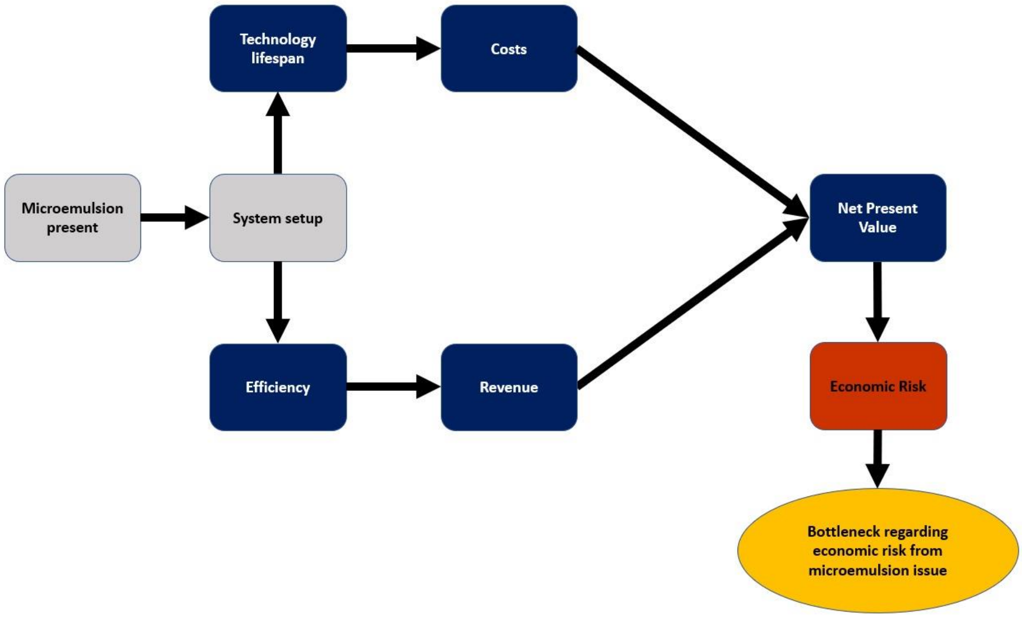
Also, by considering the system's efficiency as a fixed number and analyzing all possibilities from 0% to 100%, the software outputted that an efficiency below 24.4% approximately for extracting lithium, would compromise the system, as it would not be financially viable to implement the project.

7. FIFTH BOTTLENECK – MICROEMULSION ISSUE

The final bottleneck introduces the concept of microemulsion for the structure. A microemulsion is a thermodynamically stable fluid. Microemulsions maintain their composition, unlike kinetically stable emulsions, which eventually separate into oil and water. The particle size of microemulsions typically falls within the range of 10-300 nm. Due to the small particle sizes involved, microemulsions give the impression of being transparent or translucent solutions. Microemulsions can effectively transport various oil-soluble chemicals used in the oilfield industry. These chemicals include corrosion inhibitors, asphaltene inhibitors, scale inhibitors, and more. By utilizing microemulsions, the required quantity of organic solvent is minimized.

Moreover, microemulsions enhance the ability of oilfield chemicals to disperse within produced or pumped fluids. Consequently, this leads to improved performance of the specific chemical (FINK, 2021). To further detail this bottleneck, Figure 24 represents its diagram.

Figure 24. Fifth bottleneck, microemulsion issue.



Source: Elaborated by the author.

Therefore, applying microemulsion can obtain certain advantages for the treatment of produced water. However, observing the bottleneck, it is necessary to comprehend the advantages and disadvantages that microemulsion present in the setup can provoke for the technologies applied for direct lithium and boron removal. It can affect efficiency and technology lifespan, increasing or reducing the net present value. Consequently, microemulsion is a bottleneck for the economic risk of the structure.

To analyze the effect of the microemulsion, the previous arrangement elaborated by oil field researchers and experts (Figure 6) will be considered. Thus, a review is elaborated considering the consequences of microemulsion for electrochemical, reverse osmosis and nanofiltration treatment.

Microemulsions can have a significant impact on electrochemical processes through various mechanisms. Firstly, they can serve as a template or medium for synthesizing nanoscale electroactive materials, thereby improving the efficiency and performance of electrochemical devices. Nanoscale droplets in the microemulsion provide a large interfacial area, facilitating the rapid mass transfer and promoting more efficient charge transfer reactions. As a result, electrode kinetics are enhanced, leading to increased electrochemical activity (MACKAY et al., 2002). Additionally, microemulsions can act as carriers for active species, such as catalysts or redox mediators. Microemulsions enhance their availability and interaction with the electrochemical system by transporting and distributing these species on the electrode surface. This significantly impacts reaction rates and overall electrochemical performance, further improving the efficiency of the process (IMEL et al., 2022).

For reverse osmosis, the impact of microemulsion varies depending on the specific structure and conditions. It can have both positive and negative effects. One advantage is that microemulsion can aid in controlling membrane fouling. Microemulsion acts as a dispersant by containing tiny droplets and surfactant molecules, reducing the accumulation of contaminants and scaling materials on the membrane surface (HALLEN et al., 2021). This, in turn, improves the permeability and overall efficiency of the reverse osmosis process, which can reject a higher quantity of boron, avoiding environmental penalties. However, there are also challenges associated with microemulsion regarding membrane fouling. The composition and stability of the microemulsion can potentially lead to the formation of undesired aggregates or emulsions, resulting in fouling issues and decreased membrane performance. Therefore, it is essential to carefully formulate and optimize the microemulsion system to minimize these potential adverse effects, thus being an economic risk for directly removing lithium and boron (FOULADITAJAR et al., 2013).

Microemulsion can impact nanofiltration processes in various ways. The small droplet size and surfactant molecules in the microemulsion can enhance the separation efficiency by promoting the transport of specific solutes or ions through the nanofiltration membrane. This can be advantageous for selective separation and purification processes where targeted species must be retained or removed. Furthermore, using microemulsion as a carrier for nanoparticles or functional materials can enable the fabrication of composite membranes with improved separation properties. Incorporating these nanoparticles or functional materials into the membrane matrix can enhance the nanofiltration process's rejection efficiency, selectivity, and stability. However, it is essential to note that the stability and compatibility of the microemulsion system with the nanofiltration process need to be carefully considered. Factors such as surfactant interactions, phase separation, or fouling tendencies can affect the long-term performance and sustainability of the nanofiltration process (CHILDRESS & ELIMELECH, 1996).

8. CONCLUSION

This study presented a statistical approach to analyze 5 bottlenecks for direct lithium and boron removal from produced water. This study aimed to examine various challenges associated with removing lithium and boron from produced water and assess the feasibility of different models based on environmental and economic factors. The environmental aspect evaluates the potential risks of releasing boron into seawater, considering its hazardous nature. On the other hand, the economic analysis involves assessing the viability of extracting lithium from produced water for commercial purposes. Through Monte Carlo simulations, statistical approaches and software usage, the objective was to determine if extracting these chemicals from produced water is viable to make it an option for offshore platforms.

Therefore, to reach this goal, the 5 bottlenecks were analyzed separately and categorized by environmental or economic bottlenecks.

The first bottleneck was an environmental issue, dependent on the efficiency of the processes for removing boron (reverse osmosis and nanofiltration) and the initial concentration of the element in produced water. The results demonstrated that the pre-defined technology could disperse water in seawater at approximately 5.2 milligrams of boron per liter. Compared with countries with substantial participation in the market share for oil and gas production from offshore platforms, the results could only accommodate Saudi Arabia's limits for boron disposed in seawater (5,4 mg/L). To meet the environmental legislations of other countries, further improvements should be made in the system. For example, a 2,3% improvement in the efficiency of reverse osmosis allowed this setup to be applied in Brazil. A more significant increase of 5% and this system could also be installed in Norway. Although the hypothesis was considered, limitations were noted due to the lack of public data on boron disposal permissions for each country.

The next bottleneck was an environmental risk related to the location of geological sources and the boron concentration in produced water. Understanding the precise data for each country's boron disposal, this study could identify where this technology could be applied. For Brazil, produced water with a concentration of 19.4 mg/L or above could not be explored by this technology, resulting in environmental penalties.

The market issue is third bottleneck. Comparing lithium prices and system costs concluded that the technology was feasible, with a profit of 38,334 USD/ton of lithium. Nevertheless, the limitations of this bottleneck were noticed by the lack of precise information on electrochemistry costs (capital, operational and maintenance costs) for produced water. An analysis of other technologies currently available for removing lithium from water was selected and inserted to bypass this situation.

The next bottleneck connects with the previous one but adds the efficiency variable to avail the economic risk. Adding this variable reduced the profit to 20 440 USD/ton, approximately, which is a reduction of 46,7%. The study also displayed that the bottleneck is not financially viable if the efficiency variable is below 24,4%. Since this work is about directly removing lithium and boron, the next step could be inserting boron prices and system costs to analyze if this chemical would improve or worsen the setup.

Finally, microemulsion issue was the last bottleneck. This bottleneck was analyzed by demonstrating the positive and negative effects emulsions can have on each filtration treatment (reverse osmosis, nanofiltration and electrochemical). It was concluded that they should be considered carefully since unsolicited concentrations could have opposite effects of what is desired (membrane fouling, for example).

Therefore, this study could analyze each bottleneck separately and obtain a broader perspective on the direct removal of lithium and boron from produced water. Further studies could be done by analyzing the bottlenecks and arranging a complex structure encompassing each bottleneck's variables; environmental and economic risks could be measured in a single, more significant bottleneck. Using Bayesian networks and conditional probability could be an approach for this analysis.

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APPENDIX A – Data for first bottleneck

Value	*Frequency*	*% accumulated*
2.234055	1	0.01%
2.301301	1	0.02%
2.368546	0	0.02%
2.435792	5	0.07%
2.503037	1	0.08%
2.570283	6	0.14%
2.637528	13	0.27%
2.704773	16	0.43%
2.772019	16	0.59%
2.839264	27	0.86%
2.90651	34	1.20%
2.973755	32	1.52%
3.041001	50	2.02%
3.108246	56	2.58%
3.175492	57	3.15%
3.242737	58	3.73%
3.309983	74	4.47%
3.377228	93	5.40%
3.444474	77	6.17%
3.511719	99	7.16%
3.578965	103	8.19%
3.64621	116	9.35%
3.713455	125	10.60%
3.780701	117	11.77%
3.847946	125	13.02%
3.915192	142	14.44%
3.982437	146	15.90%
4.049683	131	17.21%
4.116928	151	18.72%
4.184174	143	20.15%
4.251419	185	22.00%
4.318665	180	23.80%
4.38591	171	25.51%
4.453156	171	27.22%
4.520401	180	29.02%
4.587647	202	31.04%
4.654892	205	33.09%
4.722137	184	34.93%
4.789383	200	36.93%
4.856628	217	39.10%
4.923874	215	41.25%
4.991119	196	43.21%
5.058365	240	45.61%
5.12561	242	48.03%

5.192856	209	50.12%
5.260101	205	52.17%
5.327347	225	54.42%
5.394592	245	56.87%
5.461838	206	58.93%
5.529083	218	61.11%
5.596328	218	63.29%
5.663574	220	65.49%
5.730819	191	67.40%
5.798065	189	69.29%
5.86531	192	71.21%
5.932556	201	73.22%
5.999801	181	75.03%
6.067047	188	76.91%
6.134292	159	78.50%
6.201538	165	80.15%
6.268783	151	81.66%
6.336029	141	83.07%
6.403274	134	84.41%
6.47052	152	85.93%
6.537765	126	87.19%
6.60501	104	88.23%
6.672256	116	89.39%
6.739501	108	90.47%
6.806747	83	91.30%
6.873992	85	92.15%
6.941238	94	93.09%
7.008483	73	93.82%
7.075729	73	94.55%
7.142974	80	95.35%
7.21022	51	95.86%
7.277465	47	96.33%
7.344711	54	96.87%
7.411956	44	97.31%
7.479202	34	97.65%
7.546447	36	98.01%
7.613692	24	98.25%
7.680938	29	98.54%
7.748183	19	98.73%
7.815429	22	98.95%
7.882674	16	99.11%
7.94992	11	99.22%
8.017165	15	99.37%
8.084411	8	99.45%
8.151656	10	99.55%
8.218902	8	99.63%
8.286147	6	99.69%
8.353393	9	99.78%

8.420638	7	99.85%
8.487884	6	99.91%
8.555129	3	99.94%
8.622374	0	99.94%
8.68962	1	99.95%
8.756865	4	99.99%
8.824111	0	99.99%
8.891356	0	99.99%
8.891367	1	100.00%

APPENDIX B – Data for first bottleneck, 5% improvement

Value	*Frequency*	*% accumulated*
2.118982	1	0.01%
2.183803	1	0.02%
2.248624	0	0.02%
2.313445	5	0.07%
2.378266	1	0.08%
2.443087	6	0.14%
2.507908	12	0.26%
2.572729	17	0.43%
2.63755	15	0.58%
2.702371	28	0.86%
2.767192	33	1.19%
2.832013	33	1.52%
2.896834	50	2.02%
2.961655	53	2.55%
3.026476	60	3.15%
3.091297	62	3.77%
3.156118	71	4.48%
3.220939	88	5.36%
3.28576	85	6.21%
3.350581	100	7.21%
3.415402	98	8.19%
3.480223	116	9.35%
3.545044	125	10.60%
3.609865	125	11.85%
3.674686	121	13.06%
3.739507	146	14.52%
3.804328	138	15.90%
3.869149	138	17.28%
3.93397	157	18.85%
3.998791	155	20.40%
4.063612	180	22.20%
4.128433	182	24.02%
4.193254	178	25.80%
4.258075	154	27.34%
4.322896	196	29.30%
4.387717	206	31.36%
4.452538	203	33.39%
4.517359	182	35.21%
4.58218	213	37.34%
4.647001	227	39.61%
4.711822	198	41.59%
4.776643	199	43.58%
4.841464	242	46.00%
4.906286	243	48.43%

4.971107	204	50.47%
5.035928	214	52.61%
5.100749	233	54.94%
5.16557	227	57.21%
5.230391	221	59.42%
5.295212	216	61.58%
5.360033	219	63.77%
5.424854	228	66.05%
5.489675	179	67.84%
5.554496	203	69.87%
5.619317	182	71.69%
5.684138	195	73.64%
5.748959	190	75.54%
5.81378	173	77.27%
5.878601	156	78.83%
5.943422	175	80.58%
6.008243	148	82.06%
6.073064	138	83.44%
6.137885	152	84.96%
6.202706	137	86.33%
6.267527	109	87.42%
6.332348	116	88.58%
6.397169	106	89.64%
6.46199	105	90.69%
6.526811	83	91.52%
6.591632	97	92.49%
6.656453	90	93.39%
6.721274	63	94.02%
6.786095	73	94.75%
6.850916	75	95.50%
6.915737	45	95.95%
6.980558	54	96.49%
7.045379	42	96.91%
7.1102	51	97.42%
7.175021	34	97.76%
7.239842	33	98.09%
7.304663	26	98.35%
7.369484	24	98.59%
7.434305	19	98.78%
7.499126	23	99.01%
7.563947	13	99.14%
7.628768	11	99.25%
7.693589	15	99.40%
7.75841	6	99.46%
7.823231	11	99.57%
7.888052	7	99.64%
7.952873	6	99.70%
8.017694	10	99.80%

8.082515	5	99.85%
8.147336	7	99.92%
8.212157	2	99.94%
8.276978	0	99.94%
8.341799	2	99.96%
8.40662	3	99.99%
8.471441	0	99.99%
8.536262	0	99.99%
8.537475	1	100.00%

APPENDIX C – Data for first bottleneck, 10% improvement

Values	*Frequency*	*% accumulated*
1.733738	1	0.01%
1.790443	1	0.02%
1.847147	0	0.02%
1.903851	4	0.06%
1.960556	3	0.09%
2.01726	5	0.14%
2.073964	10	0.24%
2.130669	14	0.38%
2.187373	19	0.57%
2.244077	27	0.84%
2.300782	29	1.13%
2.357486	30	1.43%
2.41419	44	1.87%
2.470895	64	2.51%
2.527599	59	3.10%
2.584303	64	3.74%
2.641008	73	4.47%
2.697712	85	5.32%
2.754416	103	6.35%
2.811121	98	7.33%
2.867825	94	8.27%
2.924529	128	9.55%
2.981234	117	10.72%
3.037938	143	12.15%
3.094642	128	13.43%
3.151347	148	14.91%
3.208051	137	16.28%
3.264755	144	17.72%
3.32146	193	19.65%
3.378164	154	21.19%
3.434868	190	23.09%
3.491573	170	24.79%
3.548277	195	26.74%
3.604981	166	28.40%
3.661686	211	30.51%
3.71839	223	32.74%
3.775094	195	34.69%
3.831798	198	36.67%
3.888503	205	38.72%
3.945207	228	41.00%
4.001911	238	43.38%
4.058616	202	45.40%
4.11532	229	47.69%
4.172024	221	49.90%

4.228729	232	52.22%
4.285433	241	54.63%
4.342137	225	56.88%
4.398842	229	59.17%
4.455546	235	61.52%
4.51225	204	63.56%
4.568955	217	65.73%
4.625659	198	67.71%
4.682363	198	69.69%
4.739068	194	71.63%
4.795772	208	73.71%
4.852476	177	75.48%
4.909181	168	77.16%
4.965885	168	78.84%
5.022589	164	80.48%
5.079294	175	82.23%
5.135998	141	83.64%
5.192702	115	84.79%
5.249407	151	86.30%
5.306111	125	87.55%
5.362815	122	88.77%
5.41952	91	89.68%
5.476224	81	90.49%
5.532928	100	91.49%
5.589633	96	92.45%
5.646337	93	93.38%
5.703041	75	94.13%
5.759746	72	94.85%
5.81645	54	95.39%
5.873154	51	95.90%
5.929859	45	96.35%
5.986563	50	96.85%
6.043267	47	97.32%
6.099972	42	97.74%
6.156676	31	98.05%
6.21338	28	98.33%
6.270085	19	98.52%
6.326789	24	98.76%
6.383493	26	99.02%
6.440198	13	99.15%
6.496902	11	99.26%
6.553606	9	99.35%
6.610311	6	99.41%
6.667015	11	99.52%
6.723719	8	99.60%
6.780424	5	99.65%
6.837128	9	99.74%
6.893832	9	99.83%

6.950536	6	99.89%
7.007241	4	99.93%
7.063945	1	99.94%
7.120649	1	99.95%
7.177354	1	99.96%
7.234058	3	99.99%
7.290762	0	99.99%
7.347467	0	99.99%
7.347556	1	100.00%

APPENDIX D – Data for first bottleneck, Brazil case

Values	*Frequency*	*% accumulated*
2.118982	1	0.01%
2.183803	1	0.02%
2.248624	0	0.02%
2.313445	5	0.07%
2.378266	1	0.08%
2.443087	6	0.14%
2.507908	12	0.26%
2.572729	17	0.43%
2.63755	15	0.58%
2.702371	28	0.86%
2.767192	33	1.19%
2.832013	33	1.52%
2.896834	50	2.02%
2.961655	53	2.55%
3.026476	60	3.15%
3.091297	62	3.77%
3.156118	71	4.48%
3.220939	88	5.36%
3.28576	85	6.21%
3.350581	100	7.21%
3.415402	98	8.19%
3.480223	116	9.35%
3.545044	125	10.60%
3.609865	125	11.85%
3.674686	121	13.06%
3.739507	146	14.52%
3.804328	138	15.90%
3.869149	138	17.28%
3.93397	157	18.85%
3.998791	155	20.40%
4.063612	180	22.20%
4.128433	182	24.02%
4.193254	178	25.80%
4.258075	154	27.34%
4.322896	196	29.30%
4.387717	206	31.36%
4.452538	203	33.39%
4.517359	182	35.21%
4.58218	213	37.34%
4.647001	227	39.61%
4.711822	198	41.59%
4.776643	199	43.58%
4.841464	242	46.00%
4.906286	243	48.43%

4.971107	204	50.47%
5.035928	214	52.61%
5.100749	233	54.94%
5.16557	227	57.21%
5.230391	221	59.42%
5.295212	216	61.58%
5.360033	219	63.77%
5.424854	228	66.05%
5.489675	179	67.84%
5.554496	203	69.87%
5.619317	182	71.69%
5.684138	195	73.64%
5.748959	190	75.54%
5.81378	173	77.27%
5.878601	156	78.83%
5.943422	175	80.58%
6.008243	148	82.06%
6.073064	138	83.44%
6.137885	152	84.96%
6.202706	137	86.33%
6.267527	109	87.42%
6.332348	116	88.58%
6.397169	106	89.64%
6.46199	105	90.69%
6.526811	83	91.52%
6.591632	97	92.49%
6.656453	90	93.39%
6.721274	63	94.02%
6.786095	73	94.75%
6.850916	75	95.50%
6.915737	45	95.95%
6.980558	54	96.49%
7.045379	42	96.91%
7.1102	51	97.42%
7.175021	34	97.76%
7.239842	33	98.09%
7.304663	26	98.35%
7.369484	24	98.59%
7.434305	19	98.78%
7.499126	23	99.01%
7.563947	13	99.14%
7.628768	11	99.25%
7.693589	15	99.40%
7.75841	6	99.46%
7.823231	11	99.57%
7.888052	7	99.64%
7.952873	6	99.70%
8.017694	10	99.80%

8.082515	5	99.85%
8.147336	7	99.92%
8.212157	2	99.94%
8.276978	0	99.94%
8.341799	2	99.96%
8.40662	3	99.99%
8.471441	0	99.99%
8.536262	0	99.99%
8.536768	1	100.00%

APPENDIX E – Data for first bottleneck, Norway case

Value	*Frequency*	*% accumuated*
1.873827	1	0.01%
1.933483	1	0.02%
1.993139	0	0.02%
2.052795	5	0.07%
2.11245	2	0.09%
2.172106	4	0.13%
2.231762	12	0.25%
2.291418	14	0.39%
2.351074	16	0.55%
2.41073	34	0.89%
2.470386	28	1.17%
2.530041	27	1.44%
2.589697	49	1.93%
2.649353	60	2.53%
2.709009	55	3.08%
2.768665	62	3.70%
2.828321	80	4.50%
2.887976	82	5.32%
2.947632	100	6.32%
3.007288	92	7.24%
3.066944	104	8.28%
3.1266	117	9.45%
3.186256	114	10.59%
3.245912	147	12.06%
3.305567	128	13.34%
3.365223	143	14.77%
3.424879	137	16.14%
3.484535	140	17.54%
3.544191	174	19.28%
3.603847	153	20.81%
3.663502	183	22.64%
3.723158	183	24.47%
3.782814	188	26.35%
3.84247	158	27.93%
3.902126	199	29.92%
3.961782	222	32.14%
4.021438	205	34.19%
4.081093	198	36.17%
4.140749	203	38.20%
4.200405	220	40.40%
4.260061	218	42.58%
4.319717	214	44.72%
4.379373	237	47.09%
4.439028	215	49.24%

4.498684	210	51.34%
4.55834	248	53.82%
4.617996	233	56.15%
4.677652	228	58.43%
4.737308	222	60.65%
4.796964	221	62.86%
4.856619	209	64.95%
4.916275	201	66.96%
4.975931	203	68.99%
5.035587	187	70.86%
5.095243	205	72.91%
5.154899	178	74.69%
5.214554	182	76.51%
5.27421	172	78.23%
5.333866	164	79.87%
5.393522	167	81.54%
5.453178	148	83.02%
5.512834	129	84.31%
5.57249	141	85.72%
5.632145	133	87.05%
5.691801	124	88.29%
5.751457	101	89.30%
5.811113	90	90.20%
5.870769	94	91.14%
5.930425	91	92.05%
5.99008	99	93.04%
6.049736	77	93.81%
6.109392	72	94.53%
6.169048	71	95.24%
6.228704	48	95.72%
6.28836	53	96.25%
6.348016	44	96.69%
6.407671	50	97.19%
6.467327	40	97.59%
6.526983	38	97.97%
6.586639	28	98.25%
6.646295	22	98.47%
6.705951	22	98.69%
6.765606	21	98.90%
6.825262	20	99.10%
6.884918	11	99.21%
6.944574	13	99.34%
7.00423	5	99.39%
7.063886	13	99.52%
7.123542	8	99.60%
7.183197	4	99.64%
7.242853	10	99.74%
7.302509	8	99.82%

7.362165	7	99.89%
7.421821	4	99.93%
7.481477	1	99.94%
7.541132	1	99.95%
7.600788	1	99.96%
7.660444	3	99.99%
7.7201	0	99.99%
7.779756	0	99.99%
7.779842	1	100.00%

APPENDIX F – Data for second bottleneck

Value	*Frequency*	*% accumulated*
14.5441	1	0.01%
14.65385	1	0.02%
14.76359	2	0.04%
14.87334	0	0.04%
14.98308	1	0.05%
15.09283	4	0.09%
15.20257	7	0.16%
15.31232	10	0.26%
15.42206	9	0.35%
15.53181	16	0.51%
15.64155	14	0.65%
15.7513	19	0.84%
15.86104	22	1.06%
15.97079	22	1.28%
16.08053	34	1.62%
16.19028	38	2.00%
16.30002	46	2.46%
16.40977	59	3.05%
16.51951	69	3.74%
16.62926	104	4.78%
16.739	86	5.64%
16.84875	100	6.64%
16.95849	113	7.77%
17.06824	138	9.15%
17.17798	144	10.59%
17.28773	164	12.23%
17.39747	172	13.95%
17.50722	188	15.83%
17.61696	181	17.64%
17.72671	210	19.74%
17.83645	207	21.81%
17.9462	239	24.20%
18.05594	238	26.58%
18.16569	238	28.96%
18.27543	234	31.30%
18.38518	281	34.11%
18.49493	269	36.80%
18.60467	277	39.57%
18.71442	247	42.04%
18.82416	227	44.31%
18.93391	231	46.62%
19.04365	262	49.24%
19.1534	252	51.76%
19.26314	248	54.24%

19.37289	264	56.88%
19.48263	236	59.24%
19.59238	230	61.54%
19.70212	229	63.83%
19.81187	250	66.33%
19.92161	223	68.56%
20.03136	219	70.75%
20.1411	215	72.90%
20.25085	213	75.03%
20.36059	181	76.84%
20.47034	177	78.61%
20.58008	178	80.39%
20.68983	181	82.20%
20.79957	164	83.84%
20.90932	154	85.38%
21.01906	130	86.68%
21.12881	124	87.92%
21.23855	105	88.97%
21.3483	108	90.05%
21.45804	95	91.00%
21.56779	109	92.09%
21.67753	96	93.05%
21.78728	68	93.73%
21.89702	71	94.44%
22.00677	61	95.05%
22.11651	70	95.75%
22.22626	61	96.36%
22.336	48	96.84%
22.44575	33	97.17%
22.55549	40	97.57%
22.66524	33	97.90%
22.77498	35	98.25%
22.88473	29	98.54%
22.99447	24	98.78%
23.10422	18	98.96%
23.21396	13	99.09%
23.32371	13	99.22%
23.43345	19	99.41%
23.5432	13	99.54%
23.65294	9	99.63%
23.76269	7	99.70%
23.87243	7	99.77%
23.98218	7	99.84%
24.09192	3	99.87%
24.20167	4	99.91%
24.31141	3	99.94%
24.42116	2	99.96%
24.5309	0	99.96%

24.64065	0	99.96%
24.75039	3	99.99%
24.86014	0	99.99%
24.96988	0	99.99%
25.07963	0	99.99%
25.18937	0	99.99%
25.29912	0	99.99%
25.40886	0	99.99%
25.41245	1	100.00%

APPENDIX G – Data for third bottleneck

Value	*Frequency*	*% accumulated*
-4920.45	1	0.01%
-4091.95	0	0.01%
-3263.45	0	0.01%
-2434.95	4	0.05%
-1606.45	6	0.11%
-777.948	4	0.15%
50.55144	8	0.23%
879.0511	8	0.31%
1707.551	14	0.45%
2536.05	14	0.59%
3364.55	13	0.72%
4193.05	22	0.94%
5021.549	30	1.24%
5850.049	25	1.49%
6678.549	40	1.89%
7507.048	26	2.15%
8335.548	38	2.53%
9164.047	43	2.96%
9992.547	41	3.37%
10821.05	48	3.85%
11649.55	68	4.53%
12478.05	59	5.12%
13306.55	69	5.81%
14135.05	62	6.43%
14963.54	69	7.12%
15792.04	86	7.98%
16620.54	91	8.89%
17449.04	107	9.96%
18277.54	93	10.89%
19106.04	122	12.11%
19934.54	104	13.15%
20763.04	116	14.31%
21591.54	103	15.34%
22420.04	121	16.55%
23248.54	112	17.67%
24077.04	139	19.06%
24905.54	127	20.33%
25734.04	125	21.58%
26562.54	144	23.02%
27391.04	145	24.47%
28219.54	164	26.11%
29048.04	151	27.62%
29876.54	164	29.26%
30705.04	186	31.12%

31533.54	162	32.74%
32362.04	186	34.60%
33190.54	174	36.34%
34019.04	176	38.10%
34847.54	193	40.03%
35676.04	188	41.91%
36504.54	207	43.98%
37333.04	209	46.07%
38161.53	179	47.86%
38990.03	204	49.90%
39818.53	209	51.99%
40647.03	208	54.07%
41475.53	247	56.54%
42304.03	194	58.48%
43132.53	208	60.56%
43961.03	192	62.48%
44789.53	199	64.47%
45618.03	204	66.51%
46446.53	210	68.61%
47275.03	193	70.54%
48103.53	158	72.12%
48932.03	176	73.88%
49760.53	191	75.79%
50589.03	146	77.25%
51417.53	199	79.24%
52246.03	154	80.78%
53074.53	146	82.24%
53903.03	169	83.93%
54731.53	136	85.29%
55560.03	120	86.49%
56388.53	99	87.48%
57217.03	121	88.69%
58045.53	130	89.99%
58874.03	90	90.89%
59702.53	101	91.90%
60531.02	99	92.89%
61359.52	92	93.81%
62188.02	84	94.65%
63016.52	60	95.25%
63845.02	74	95.99%
64673.52	56	96.55%
65502.02	53	97.08%
66330.52	48	97.56%
67159.02	40	97.96%
67987.52	44	98.40%
68816.02	39	98.79%
69644.52	31	99.10%
70473.02	20	99.30%

71301.52	23	99.53%
72130.02	15	99.68%
72958.52	6	99.74%
73787.02	12	99.86%
74615.52	2	99.88%
75444.02	8	99.96%
76272.52	0	99.96%
77101.02	2	99.98%
79351.46	2	100.00%

APPENDIX H – Data for fourth bottleneck

Value	*Frequency*	*% accumulated*
-12752	1	0.01%
-11961.6	0	0.01%
-11171.3	2	0.03%
-10380.9	1	0.04%
-9590.54	1	0.05%
-8800.19	4	0.09%
-8009.83	4	0.13%
-7219.47	6	0.19%
-6429.11	11	0.30%
-5638.75	17	0.47%
-4848.39	18	0.65%
-4058.03	21	0.86%
-3267.67	21	1.07%
-2477.31	37	1.44%
-1686.96	44	1.88%
-896.596	62	2.50%
-106.237	75	3.25%
684.1218	66	3.91%
1474.481	92	4.83%
2264.84	79	5.62%
3055.199	93	6.55%
3845.558	123	7.78%
4635.917	121	8.99%
5426.276	133	10.32%
6216.635	147	11.79%
7006.994	155	13.34%
7797.353	191	15.25%
8587.712	186	17.11%
9378.071	185	18.96%
10168.43	220	21.16%
10958.79	211	23.27%
11749.15	200	25.27%
12539.51	220	27.47%
13329.87	234	29.81%
14120.22	238	32.19%
14910.58	249	34.68%
15700.94	282	37.50%
16491.3	208	39.58%
17281.66	239	41.97%
18072.02	246	44.43%
18862.38	241	46.84%
19652.74	264	49.48%
20443.1	257	52.05%
21233.46	231	54.36%

22023.81	252	56.88%
22814.17	235	59.23%
23604.53	230	61.53%
24394.89	224	63.77%
25185.25	229	66.06%
25975.61	238	68.44%
26765.97	225	70.69%
27556.33	200	72.69%
28346.69	185	74.54%
29137.05	189	76.43%
29927.4	189	78.32%
30717.76	191	80.23%
31508.12	147	81.70%
32298.48	157	83.27%
33088.84	143	84.70%
33879.2	120	85.90%
34669.56	134	87.24%
35459.92	137	88.61%
36250.28	110	89.71%
37040.64	83	90.54%
37830.99	100	91.54%
38621.35	92	92.46%
39411.71	83	93.29%
40202.07	78	94.07%
40992.43	85	94.92%
41782.79	55	95.47%
42573.15	50	95.97%
43363.51	40	96.37%
44153.87	40	96.77%
44944.22	38	97.15%
45734.58	39	97.54%
46524.94	47	98.01%
47315.3	28	98.29%
48105.66	22	98.51%
48896.02	13	98.64%
49686.38	18	98.82%
50476.74	21	99.03%
51267.1	22	99.25%
52057.46	9	99.34%
52847.81	7	99.41%
53638.17	8	99.49%
54428.53	8	99.57%
55218.89	6	99.63%
56009.25	8	99.71%
56799.61	1	99.72%
57589.97	2	99.74%
58380.33	8	99.82%
59170.69	3	99.85%

59961.05	5	99.90%
60751.4	7	99.97%
61541.76	1	99.98%
62332.12	1	99.99%
63122.48	0	99.99%
63912.84	0	99.99%
64703.2	0	99.99%
65493.56	0	99.99%
66325.64	1	100.00%